

# **ACTIVITIES FOR THE SCHOOL MATHEMATICS LAB**

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## Acknowledgement

This report describes the activities, games and puzzles developed for the mathematics laboratory at HBCSE by the authors. The authors have relied for ideas on the sources mentioned in the report. We gratefully acknowledge this contribution.

We thank Jacob, Ahire, Rane, Dhruva, Rajesh and Shekhar for help in making the items for the mathlab.

We thank Shekhar Agre for extensive help in typing parts of the draft of this report and for drawing on the computer. We also wish to thank Porus, Kala, Dr. G.C. Pal and Dr. Neelima Srivastava for helping with preparing the report. We are indebted to N.S. Thigale and Gajanan Mestry for completing the xeroxing and binding with speed and accuracy.

The work on the mathlab and the report was completed during K.M. Bhatt's stay at HBCSE as a teacher fellow between December 1996 and April 1997. This author wishes to thank HBCSE, its director Dr. Arvind Kumar and Dr. H.C. Pradhan for all the facilities made available during the tenure of the fellowship. Bhatt also expresses his gratitude to other colleagues at HBCSE for their friendly and co-operative interaction.

The Atomic Energy Education Society made it possible for K.M. Bhatt to avail the fellowship at HBCSE and we are grateful to the Society for this. K.M. Bhatt also wishes to thank Dr. S.P. Katuria, secretary, AEES and Dr. C.L. Thaper, former secretary, AEES for their encouragement and help. He also wishes to thank Mrs. Rani Verma, Principal, AECS-2, Mumbai for her encouragement and opportunity to present his work to colleagues. Bhatt also wishes to thank the Principal of A.E.C.S. school at Hyderabad, Mr. J.V. Mohan Rao, for providing leave to avail the fellowship.

K.M. Bhatt wishes to thank his wife Anjali for shouldering all domestic responsibilities during the fellowship period. K. Subramaniam wishes to thank the members of his family for putting up with converting a part of the home into a DTP unit.

*HBCSE, Mumbai*

**K.M. Bhatt**  
**K. Subramaniam**

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# Introduction

Many students in our schools even today show a kind of displeasure to the learning of mathematics. This happens mainly because of the method of teaching that we teachers usually adopt in the classroom. Often it is dull, boring, and stereotyped. Some students even think that mathematics means to 'get something right' or to 'get something wrong'. When they get it wrong they think that they are not good enough for mathematics and lose interest in learning. On the other hand a few students learn enough tricks to 'get something right' to please the teacher and pass the exam.

There are not enough instances when a teacher has tried to teach maths in a different way, say through activities and games. These activities and games make the process of learning more interesting and effective. The setting up of a mathematics laboratory is an attempt in this direction.

A mathematics laboratory is a place where in we find a collection of games, puzzles, teaching aids and other activities. These activities are used both by the teacher and the students to explore the world of mathematics, to discover, to learn and to develop an interest in mathematics. Although mathematics is not an experimental science in the way in which physics, chemistry and biology are, a mathematics laboratory can contribute greatly to the learning of mathematical concepts and skills.

We have tried to list the different ways in which we feel that a mathematics laboratory could contribute to learning mathematics:

- A mathematics laboratory provides an opportunity for the students to discover through doing. In many of the activities, students learn to deal with problems in the course of doing concrete activity which lays down a base for more abstract thinking.
- It gives more scope for individual participation. It encourages students to become autonomous learners and allows a student to learn at his or her own speed.
- It widens the experiential base, and prepares the ground for later learning of new areas in mathematics and of making appropriate connections.
- In various puzzles and games, the students learn the use of rules and constraints and have an opportunity to change these Rules and constraints. In this process

they become aware of the role that rules and constraints play in mathematical problems.

- Because of the larger time available individually to the student and opportunity to repeat an activity several times, students can *revise, resee and rethink* the problem and solution. This helps to develop metacognitive abilities.
- It builds up interest and confidence in the students in learning and doing mathematics.
- Importantly, it allows variety in school mathematics learning.

In this report we have described some activities which could typically be included in a school mathematics laboratory. The activities are suitable for students of class 6 to class 10. We have also included a couple of activities suitable for a lower level -- the *place-value snake* and the *fraction chart*. The items have been grouped under two broad headings: **activities** and **games and puzzles**. The activities could be done individually by students, with guidance from a teacher, or could be used for demonstration to a small group of students. Some of the activities could also be used as teaching aids in a classroom. The games and puzzles are fun to do individually and all of them contain some element of mathematics which can be explored while doing them or as a sequel. Some of these items have been developed by the authors at HBCSE. Others have been taken from articles and books and have been modified or developed further. A list of these sources appears at the end of the description of activities. In addition, we have included, as an appendix, an annotated compilation of more activities from other sources, which are suitable and interesting for inclusion in the mathematics laboratory. Some of these need to be modified or developed further and we hope to be able to take this up soon at HBCSE.

# PART I

## Activities

### 1 Area and Perimeter

*Materials required: Square 10"  $\times$  10" board of plywood or hardboard or acrylic, matchsticks or short nails, string.*

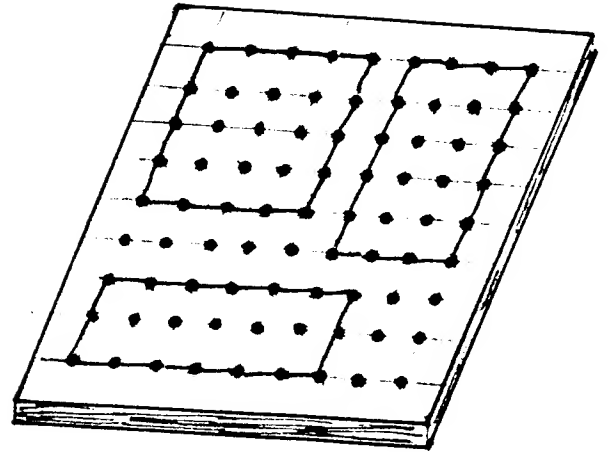
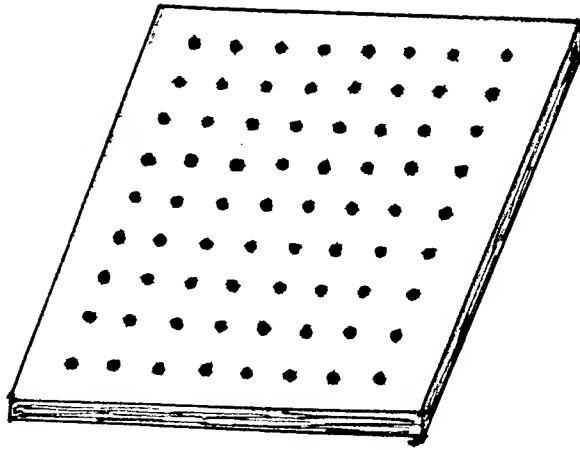
Many high school students find it difficult to master the concept of perimeter even though it is a simple concept. Further, its relation to area, which is the other measure associated with a bounded plane figure, is not clear. Some students even have difficulty with the concept of area. This activity clarifies the notion of perimeter and helps students discover the independence of area and perimeter.

Drill holes on the square board at 1 inch intervals forming a square grid. The holes need to be just large enough to easily fit pegs which is about 2mm in diameter. If the board is made of transparent acrylic sheet can also be used on the OHP. Another advantage with an acrylic sheet is that, it can be placed on top of a graph paper, which can be used to estimate areas. (Materials such as thermocol, or chappal sole rubber can also be used if they are available easily.) Break the match sticks in half and scrape off the phosphorous for using them as pegs on the board. Alternatively short nails with small heads can be used. In all about a dozen pegs may be required.

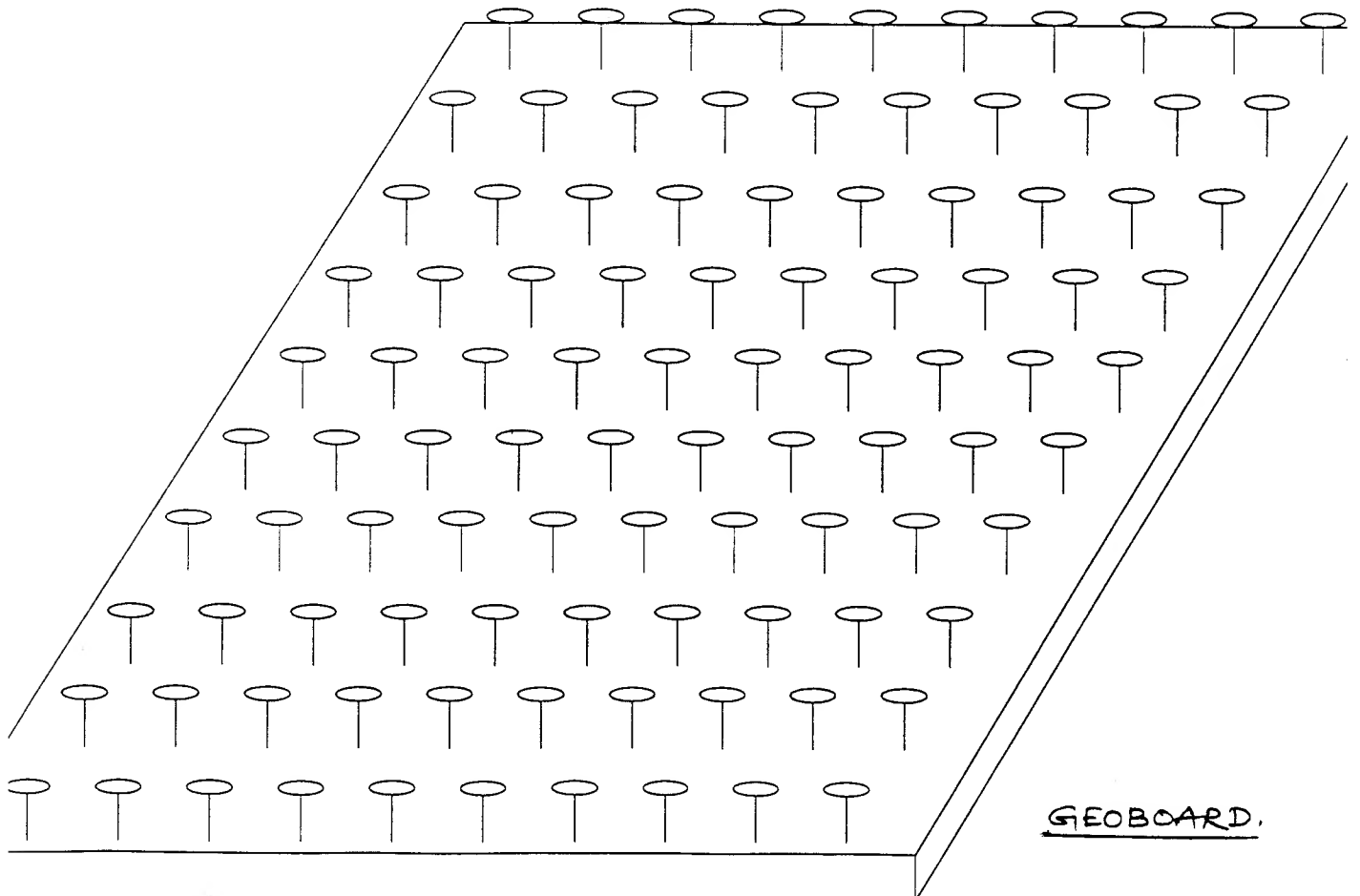
The following activities can be performed with the pegboard:

- Take a length of string about a foot long and knot its two ends together to form a loop. Give the loop to students along with four pegs. What are the different kinds of figures which they can make? Ask them to name the figures. Ask them what remains the same as these figures change. Introduce the concept of perimeter as the total length of the boundary.
- Take a total string length of 20 inches which is knotted into a loop and four pegs. Ask the children to make rectangles of different sizes. Which rectangle has the largest area?
- With the same loop of string, ask the students to form figures with 3 pegs, with 5 pegs, 6 pegs and so on. Can they estimate the area? Which figure has





PEG BOARD.



GEOBOARD.

the largest area?

- Ask students to construct figures with a given area and measure the perimeters. Which figure has the largest perimeter?

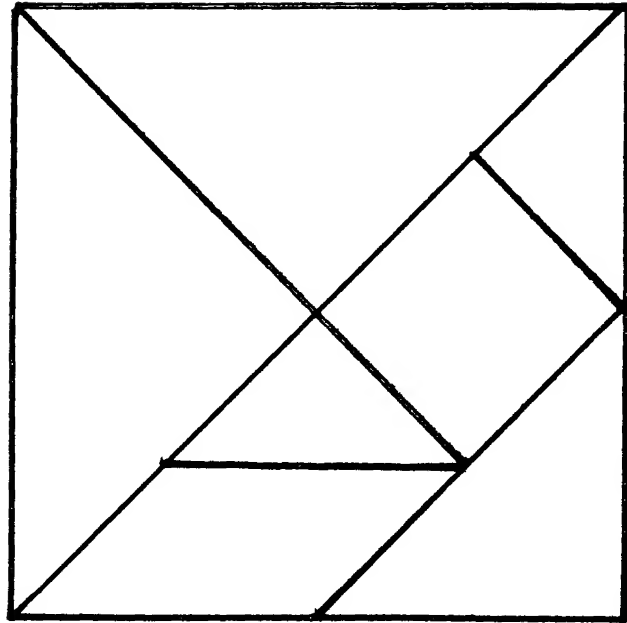
## 2 Geoboard

*Materials required: Square board 10"  $\times$  10" of wood or acrylic, nails or pegs. superglue if acrylic board is used.*

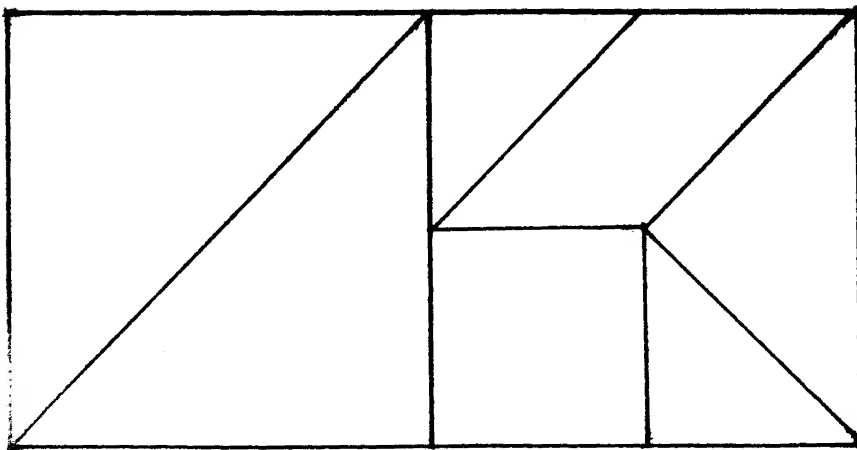
A geoboard is a very useful device for introducing children to important topics in school geometry. It consists of an array of nails or pegs which are placed at equal distances on a square acrylic board making up a grid. The baseboard is made from wood or acrylic. Drill holes at equal distances forming a square grid. It is convenient to place the pegs at gaps of 2 cm. Nails can also be used, but the heads must be small. Acrylic boards with holes drilled in them are good for use on an overhead projector. Graph paper can also be placed below an acrylic board which is useful in estimating areas. The pegs or nails can be glued onto the acrylic board.

Polygonal figures can be made on the geoboard by stretching rubber bands across the pegs. A number of activities which are suitable for students of different classes can be performed on the geoboard. Some of the activities that can be performed are:

- Study of kinds of angles.
- Areas and perimeter
- Property of similar figures: Take two rubber bands and form the triangles one within the other. Are they similar? Why or why not?
- On the geoboard it is very easy to find areas of figures by completing rectangles. Hence it is easy to verify theorems about areas. For example, with two rubber bands the theorem, "the area of a triangle on the same base and between the same parallels is half that of the parallelogram" can be verified.
- Another activity on the geoboard is to construct right angle triangles such that none of the sides of the triangle is aligned along a row or a column. How many such triangles can be constructed? What is the relation between the sides of these triangles?

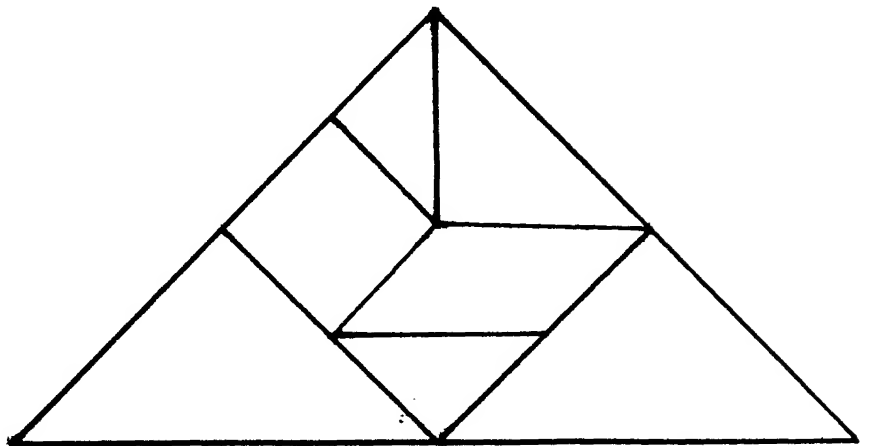


TANGRAM.



TANGRAM PIECES  
REARRANGED TO  
GET A RECTANGLE.

TANGRAM PIECES  
REARRANGED TO  
GET AN ISOSCELES  
TRIANGLE



- An interesting activity to do with the geo-board is to discover the Pick's theorem. The steps for this activity are described briefly:
  - Make different polygonal shapes on the geoboard. Can you determine the areas of these polygonal figures?  
There is a simple formula for obtaining the area of a polygonal figure on the geoboard. It is not difficult to derive this formula. Try to!
  - First begin with a simple figure that is easy — the unit square. What happens to the square as you deform one of its four sides, keeping its other three sides intact? Make a change in one of the sides of the square and describe the result by answering the questions below:
    - \* What is happening to the number of boundary points? Is it increasing or decreasing or remaining the same?
    - \* What is happening to the number of points in the interior the polygon? Is it increasing? Are any of the boundary points becoming interior points?
    - \* Prepare a table for each step as you change one side of the square. Similarly start with a rectangle and observe what happens. Can you arrive at a general rule for the area of a polygon?

The general rule for areas of polygonal figures is given by the formula  $A = \frac{B}{2} + I - 1$ .  $A$  is the area of the figure,  $B$  the number of boundary points, and  $I$  the number of interior points.

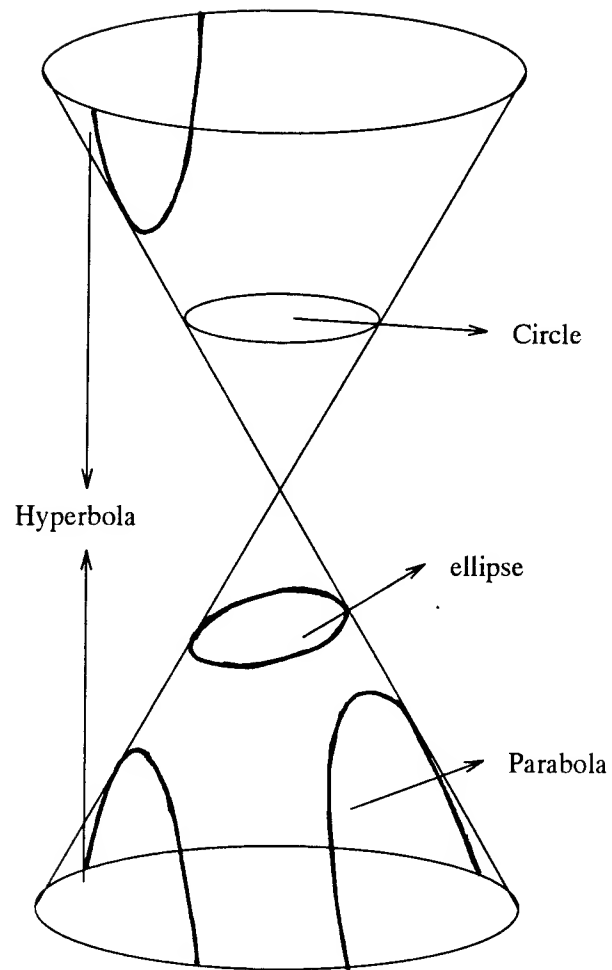
### 3 Tangrams

*Materials required: Card paper.*

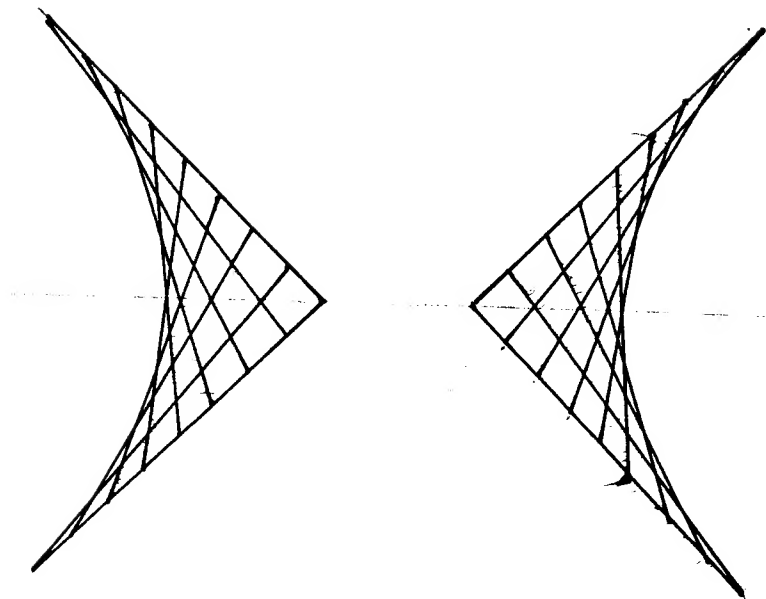
A Tangram set has seven pieces which are cut from a square piece. The shape and pattern of these pieces is shown in the figure. Tangram puzzles were first developed by Chinese over a thousand years ago. Tangrams are excellent for children to work with in developing concepts of geometric relations.

To make the tangram, cut a square piece of card paper (about 10" x 10"). Cut it in smaller pieces as shown in figure. The tangram pieces can be arranged to form hundreds of different shaped animal and human figures.

Tangrams can also be used to study geometric shapes. Some activities which can be performed using tangrams are the following:



### CONIC SECTIONS



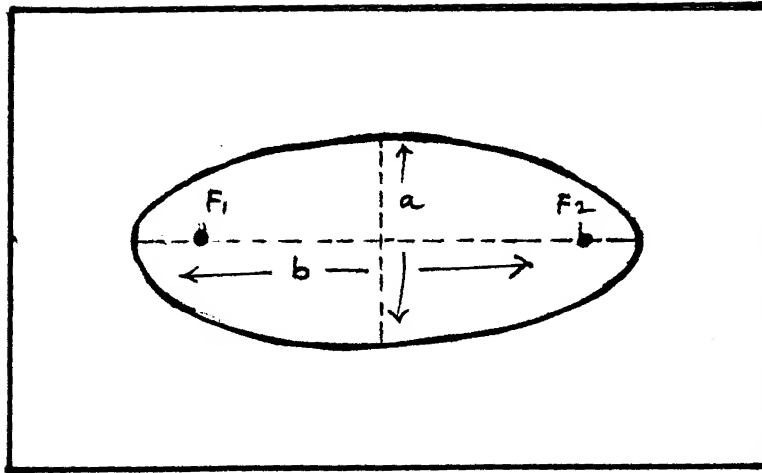
### HYPERBOLA.

- List the different kinds of figures present in the tangrams. How many pairs of equilateral triangles are there in the set?
- What is the area of each tangram piece and of the whole tangram set taking the area of the smallest triangle as one unit?
- Take two small triangles and put them together in such a way that we get a square, a triangle and a parallelogram which are all equal in area.
- Repeat the above activity with two big triangles.
- Make a square, a triangle and a parallelogram using two small triangles and a medium sized triangle.
- Make a rectangle and a trapezium using two small triangles and a parallelogram.
- Make a rectangle, a trapezium and a parallelogram using two small triangles and a square.
- Make a square, a triangle and a parallelogram using two small triangles, one medium sized triangle and a big triangle.
- Make a square, rectangle and a triangle using all seven pieces.
- Find the area of a given combination of figures taking the area of smallest triangle as one square unit.
- Proof of theorems such as the midpoint theorem, properties of a parallelogram can be discussed using tangram pieces.

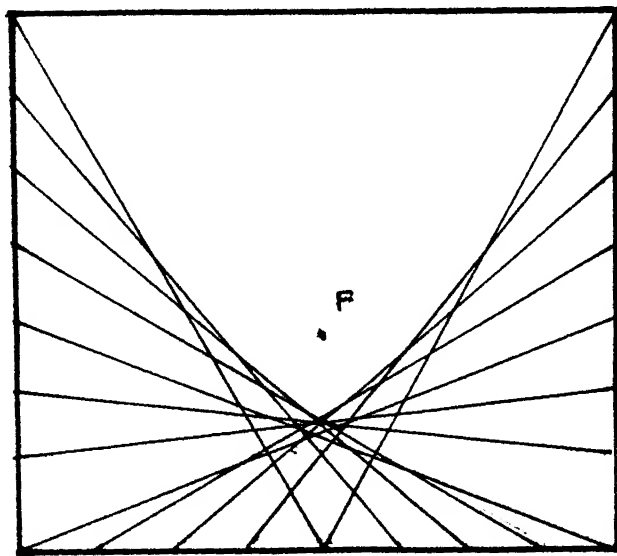
## 4 Conic Sections

*Materials required: Modelling clay, chart paper, tracing paper, thin wire, divider, plane paper, pen, wooden board, nails string, filter paper, drawing pins, thread.*

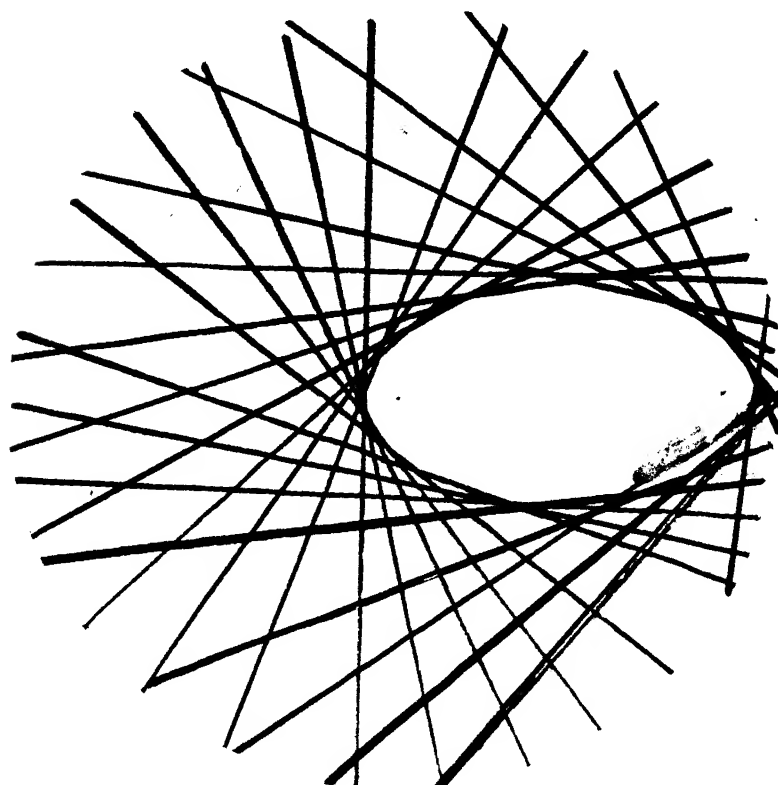
By cutting through a cone at different angles to the base we can produce a family of interesting geometric curves are called conic sections. These curves are circle, parabola, hyperbola and ellipse.



ELLIPSE.



PARABOLA BY  
PAPER FOLDING.



ELLIPSE BY  
PAPER FOLDING

## Conic sections using modelling clay

If a hollow cone is cut open by cutting along its slant edge and unrolled, we get a sector of a circle. Hence a cone can be rolled from a portion of circular paper. Cut a piece of chart paper into a circle using a divider. Cut away approximately a quarter of a circle by cutting along radial lines, retaining three quarters of it. This can be rolled into the shape of a cone, and the overlapping portion can be pasted or stapled together. Now make an identical cone in the same way from butter paper or tracing paper, but do not staple or paste it into the final shape. Place it inside the chart paper cone to form an inside layer. Now fill the cone with modelling clay. Press it down till it take a shape of a cone. Let the clay cone with the butter paper cover fall out of the paper cone. Peel off the butter paper. The cone is now ready for cutting.

Stretch a thin wire holding the ends with both hands till it is taut. Use this wire to cut the cone at different angles to the base.

- A cut parallel to the base gives a circle at the section.
- A cut parallel to a sloping side gives a parabola.
- An ellipse is made by cutting through the cone at a slant.
- A cut parallel to the axis of the cone gives a hyperbola.

## Drawing an ellipse on board with two pegs or nails

A simple way to produce an ellipse is to use a thread and two nails. First hammer the two nails at certain distance. Now form a loop from the string about three times as long as the distance between the nails and place it across the nails. Now put the point of the pencil inside the loop and move it around the pins, keeping the pencil slanted so that the line is as smooth as possible. The curve that is obtained is an ellipse. Here the two nails form the foci of the ellipse. Ask the students why the sum of the distances from the two foci of every point on the ellipse is constant.

## Conic sections through paper folding

One can obtain a beautiful ellipse by folding a circular piece of paper like the filter paper. Mark any point in the interior of the filter paper (preferably about 1 cm from the edge). Now fold the paper so that the circumference falls on F and press it down to crease it. Let the point on circumference which coincides with F be  $P_1$ . Take



the adjacent point on the circumference  $P_2$  (say about 1 cm away from  $P_1$ ) and fold the paper again so that  $P_2$  falls on F. Crease the paper once again. Similarly fold the paper repeatedly so that a large number of points on the circumference fall on F. The whole circumference must be covered in this way. Examine the pattern of creases obtained. A beautiful ellipse appears in the middle as an envelope of all the creases. The point F and the center of the circle form the two foci of the ellipse.

A parabola can be obtained from a rectangular piece of paper in a similar manner. Mark a point F near one of the edges of the paper. Now fold repeatedly so that the edge close to F falls on F. The pattern of creases that emerge show a parabola. The point F forms the focus of the parabola and the edge which falls on F while folding forms the directrix of the parabola.

To fold a hyperbola take a piece of tracing paper (or some transparent paper which is not too thin and which creases well). Draw a circle on the paper and mark a point F outside the circle. Fold the paper repeatedly so that F falls on different points on the circumference of the circle. A pattern of two hyperbolas is obtained each of which is a reflection of the other. The center of the circle and the point F form the two foci of the hyperbola.

## Making a parabola with drawing pins and thread

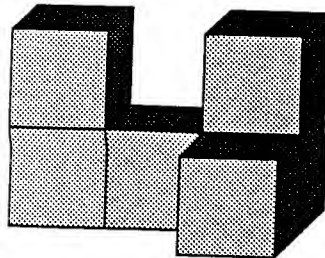
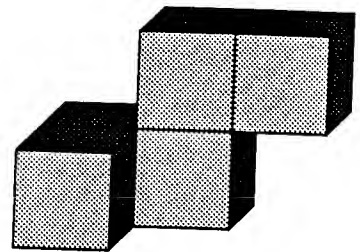
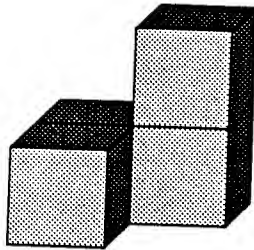
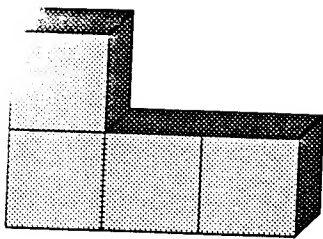
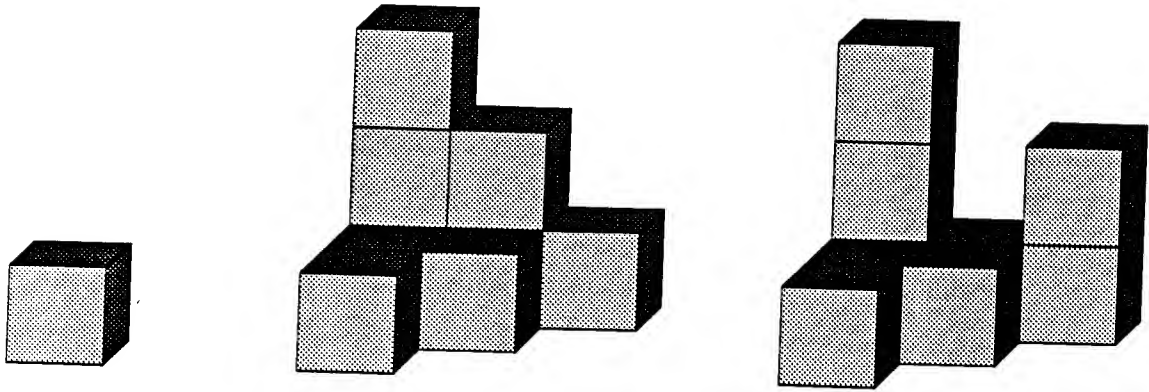
We can make a hyperbola with thread and drawing pins. In this activity we push about six drawing pins into a piece of cardboard so that they lie at equal distances on a straight line. Make another identical line of drawing pins with the same gaps as the first line. The two lines must intersect to make an angle. Now with a piece of thread join the first pin of one line to the last (sixth) pin of the other to form a line segment between the pins. Join the second pin of the first line to the fifth pin of the second line. Continue joining the pins in this manner to obtain the hyperbola shown in the figure.

## Exercise

In the paper folding activity for constructing conic sections, proof of the following can be given as exercise to the students:

1. The sum of the distances of any point on the ellipse from the two foci is constant.

**Hints:** Each of the folded creases is a tangent to the ellipse. Study what



CUBES WITH DIFFERENT SPACIAL ORIENTATIONS

happens with a single tangent. Folding creates a reflection with respect to the folded line of point F on the circumference. An important step is to identify the point on the tangent which touches the ellipse.

2. The sum of the distances of any point on the parabola from the focus and the directrix is constant.
3. The difference of the distances of any point on the hyperbola from the two foci is constant

## 5 Surface area and volume

*Materials required: eight or more wooden or plastic cubes or alternatively chart paper from which cubes can be made.*

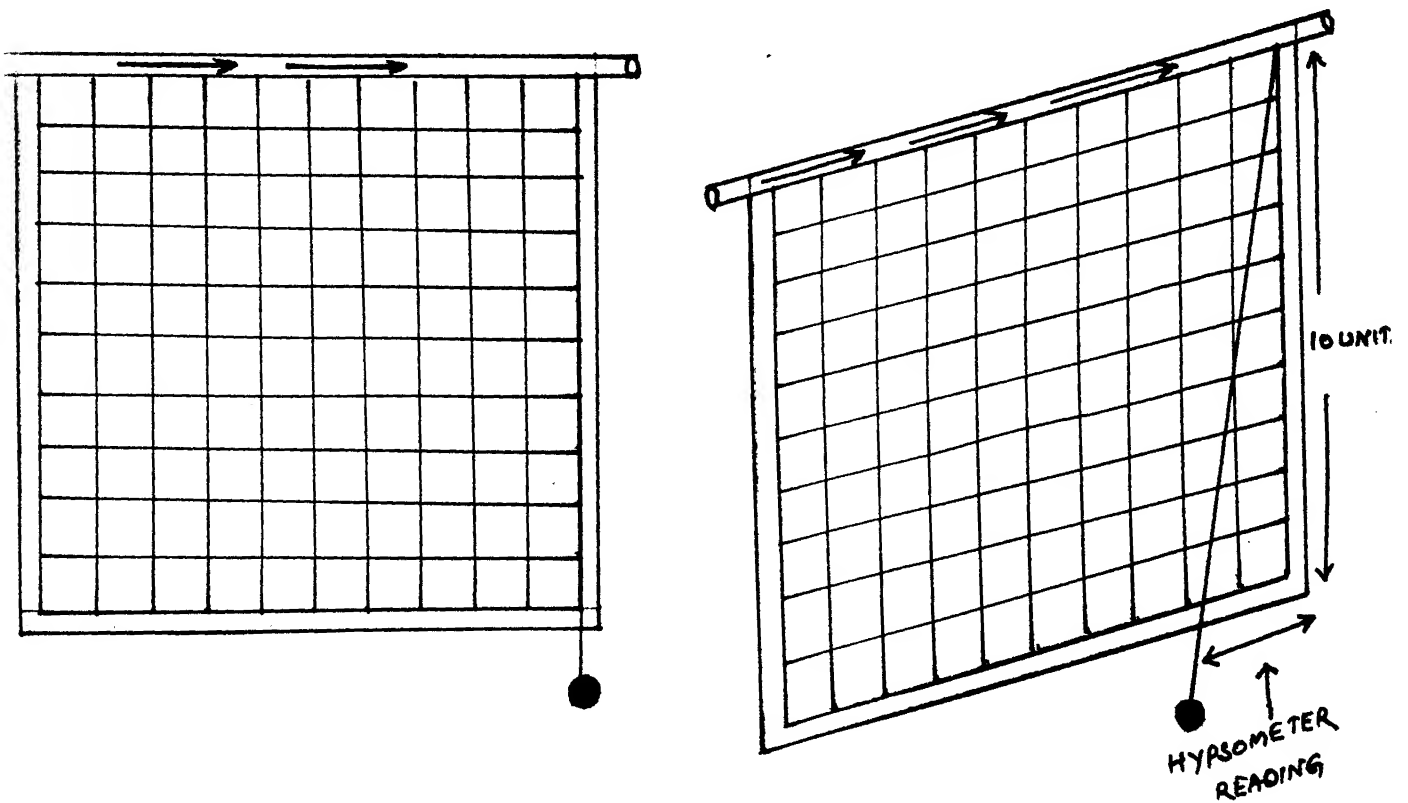
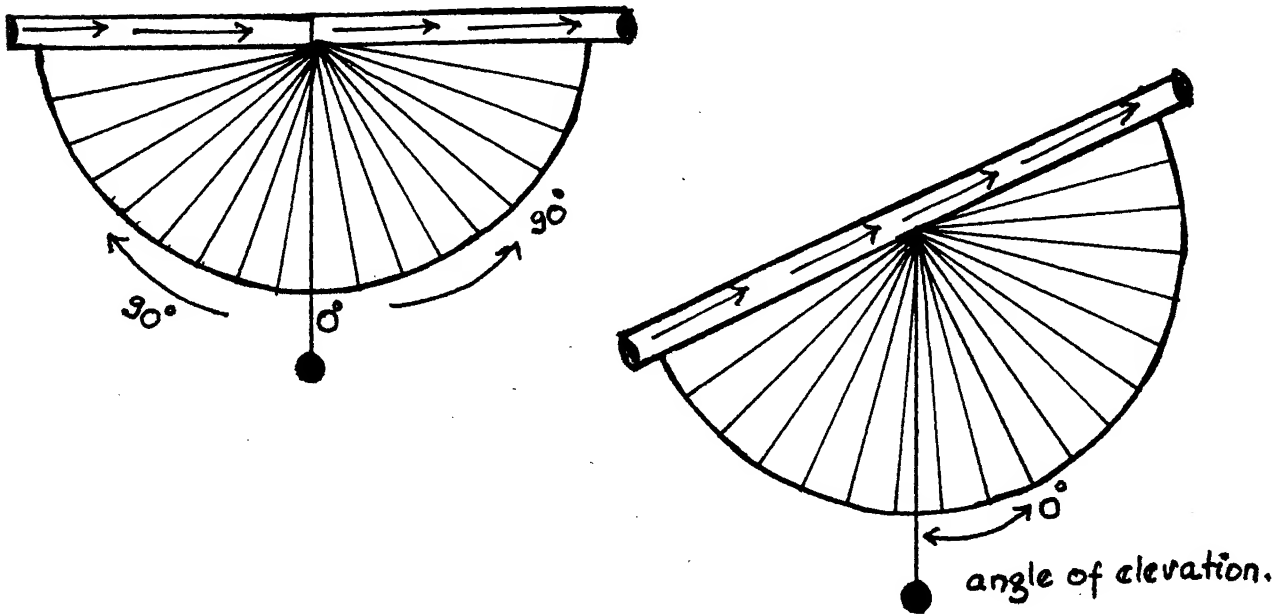
In the school curriculum students are introduced to solid geometry through the topic of mensuration. They need to compute the surface area and volume of a number of regular solids. Often students have not fully grasped these concepts of surface area and volume or appreciated the difference between the two. In this activity students compare the surface area and volume of shapes formed by piling identical cubes in different ways. The activity can also be extended to explore spatial relationships using the cubes.

For this activity we need eight or more identical cubes of side 1 cm (or 1 inch or of suitable side taken to be 1 unit). The volume of each cube is then one cubic cm. (or one cubic unit). The surface area of each cube is one square cm. (or one square unit) and the total surface area as  $6 \times 1 = 6$  sq.cm. (or 6 sq units). If wooden or plastic cubes are not available, the cubes can be made from chart paper or from a thin block of soft wood. If chart paper is used, one can make cubes of five sides, with one side open which are easy to make. Five sided-cubes are sufficient for the activity.

The following activities may be done with the cubes:

- Ask the students to first find out the total volume and area of all the right cubes.
- Give the students five cubes and ask them to arrange in all possible ways. The cubes must be arranged so that when two cubes make contact, the faces in contact must completely cover each other. For each arrangement let them find the surface area and volume.

## ASTROLABE.



## HYPSONETER.

- Repeat this activity with 6, 7 and 8 cubes. Find each time the surface area and volume.
- While making each of the arrangements with the cubes the students can be asked to project the plan and the elevations from different sides of the solid figure.
- Given the two dimensional drawings of the plan and elevation, other students can be asked to build up the arrangements using the cubes.

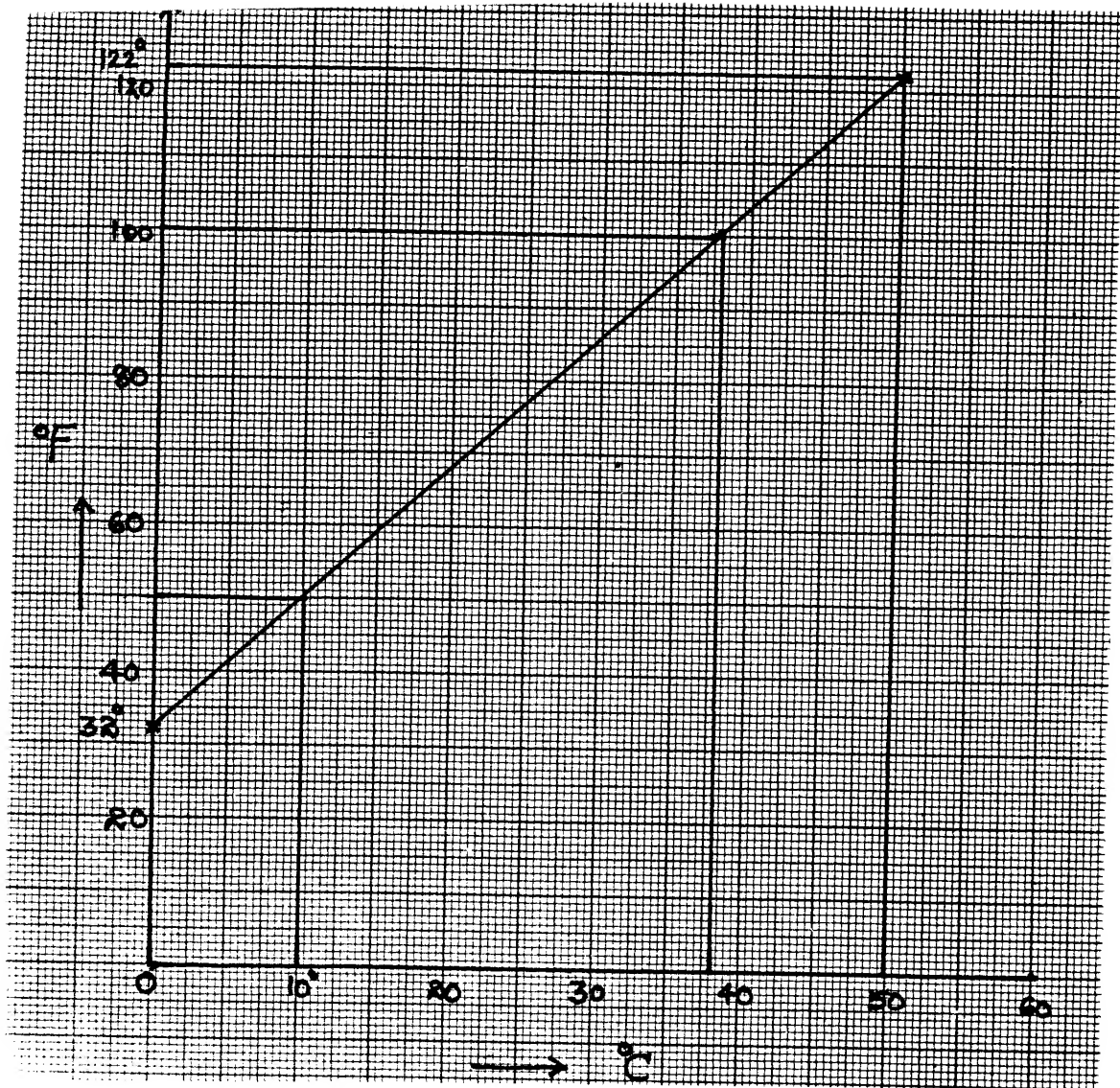
## 6 Astrolabe

*Materials required: Chart paper, pen, divider, piece of string, a small heavy object to be tied as a bob to the string, hollow tube made of paper or curtain rod about 10 inches long.*

An astrolabe is an instrument used commonly in ancient and medieval astronomy to find the angular distances of stars and planets. It can also be used to find the angle of elevation of mountains, trees, buildings or other tall objects.

You can make an astrolabe from a chart paper cut into a semi-circle of about 5 inch radius. A divider or a pair of scissors can be used to cut the chart paper in the form of a semi-circle. Mark the angles from  $0^\circ$  to  $180^\circ$  on the semi-circle similar to the markings on a protractor. Roll a rectangular piece of chart paper into a hollow tube and stick it with cello tape to prevent it from unrolling. Attach the paper-tube, or if it is available a hollow curtain rod, along the diameter of the semi-circle. From the centre of the circle (which is also the centre of the hollow tube) suspend a plumb-line (bob tied to the string) as in the figure.

To find the angle of elevation of a tall object, look through the tube at the top of the object. The angle made by the plumbline with the  $90^\circ$  line on the semi-circle is the angle of elevation of the object. If this angle is  $\theta$ , then the height of the object above eye-level can be obtained by multiplying the distance of the object by  $\tan \theta$ .



CONVERSION CHART.

## 7 Hypsometer:

*Materials required: cardboard, graph paper(optional), hollow chart paper tube or curtain rod about 10 inches long, string and small heavy object to be tied to the string as a bob.*

This is a modification of the astrolabe, which uses the property of similar triangles to estimate heights of tall objects. With this change it becomes much easier to estimate the heights since we do not use trigonometric ratios. So it can also be used by a student who is not yet introduced to trigonometry.

To make the Hypsometer first cut the cardboard into a rectangle of size 10" x 11". Paste a sheet of graph paper in inches or plain paper on the cardboard. Draw a 10" x 10" grid on the cardboard as shown in the figure. Graduations in tenths of an inch are marked on the bottom line of the grid. (This is not necessary if one uses a graph paper with inch squares.) Now fix the hollow tube at the top on the first row and suspend the string with the bob from the tube at the first point, that is at the right hand top corner of the grid, as in the figure.

To use the hypsometer, hold it parallel to the ground such that the string covers the first line (which therefore becomes vertical). Now looking through the tube point it to the top of the object whose height is to be estimated. Ask your friend to note the reading where the string crosses the bottom line of the grid drawn on the cardboard. This is the distance from the right most vertical line and is the Hypsometer reading. Now using the following formula we can find the height of the object from your eye line:

$$height = \frac{\text{hypsometer reading}}{10} \times \text{distance of object}$$

Add your height to this, which gives the height of the object from the ground.

## 8 Conversion chart:

*Materials required: Graph sheet and pen.*

Students in the high school often need to convert temperature measurements from the Celsius scale to the Fahrenheit scale and vice versa. This activity aims at making a chart for converting from one scale to another.

Ask the students to plot the different scales on the X and Y axis on a graph paper. Let them obtain the conversion for three different values of temperature using the

1	2	3	4	5	6	7	8	9
				1	1	1	1	1
2	4	6	8	0	2	4	6	8
3	6	9	2	5	8	1	4	7
4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5
6	2	8	4	0	6	2	8	4
7	4	1	8	5	2	9	6	3
8	6	4	2	0	8	6	4	2
9	8	7	6	5	4	3	2	1

NAPIER'S BONES.



formula  $C = \frac{5}{9}(F - 32)$ . Ask them whether the three points lie on a straight line. Let them draw a straight line through these points. Are the temperatures of the ice point and steam point shown correctly in both degree Celsius and degree Fahrenheit by the straight line? Ask the students to verify whether the line can be used as a chart to convert any measure of temperature from Celsius to Fahrenheit. Why is this possible?

The chart provides an occasion to discuss the property of linear functions. Let the students explore making charts of other linear functions such as converting inches to cms, or a chart for finding the father's age from the son's age and so on.

## 9 Napier's bones

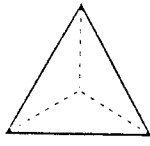
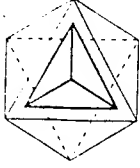
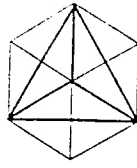
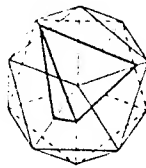
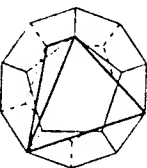
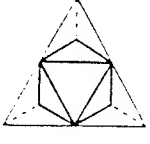
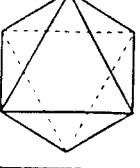
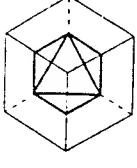
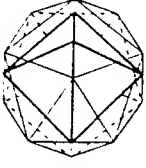
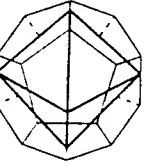
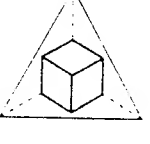
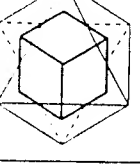
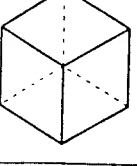
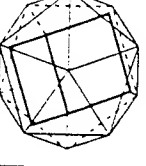
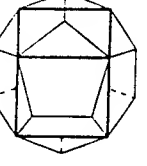
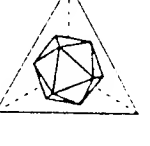
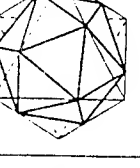
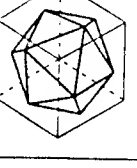
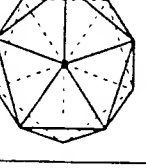
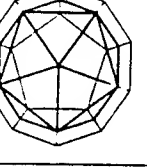
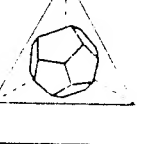
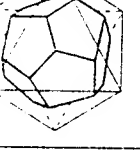
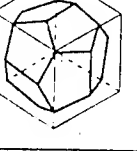
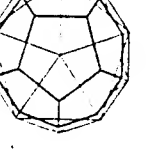
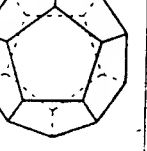
*Materials required: Cardboard or chart paper.*

John Napier (1550-1617), the man who invented the logarithmic tables also made this simple calculation device. Napier's bones can be used for multiplying a large number with a single digit number. In this way it reduces multiplication to a series of additions.

Take nine long strips of chart paper or cardboard, 1 inch wide and 9 inches long. Copy the numbers shown in the figure on the nine strips in the manner shown along with the diagonal lines. Now, to multiply 2356 by 8, take the bones with the digits 2, 3, 5 and 6 and lay them next to each other. On the 8th horizontal row add the digits that are formed along each diagonal to get each digit of the answer.

The method of adding along the diagonals used here is very similar to the **Gelosia method** of multiplication developed by Indian mathematicians many centuries ago. It is also quite similar to the method of multiplication taught at school. On observing closely one finds that each of Napier's bones is simply the multiplication table for that number (verify this from the figure). The digits in the units place and the tens place are written separately. So while using Napier's bones, one need not make the effort of recalling multiplication tables. Addition of digits, carrying over the digit in the tens place are managed by the diagonal arrangement.

# Relationships between Pairs of Platonic Polyhedra

		Outer Figures				
		Tetrahedron	Octahedron	Cube	Icosahedron	Dodecahedron
Inner Figures	Tetrahedron					
	Octahedron					
	Cube					
	Icosahedron					
	Dodecahedron					

(SOURCE : THE MATHEMATICS TEACHER, VOL : 87, MAY 1994)

## 10 Nested Platonic solids

*Materials required: Drinking straws preferably the harder variety, rounded paper clips, needle and thread.*

A platonic solid is a solid whose faces are regular polygons. All its faces are congruent, that is all its faces have the same shape and size. Also all its edges have the same length. Platonic solids are regular *tetrahedra*.

The most common platonic solid is the **cube**. It has six faces and each face is a square. Another platonic solid is the regular **tetrahedron** which has four faces and each face is an equilateral triangle.

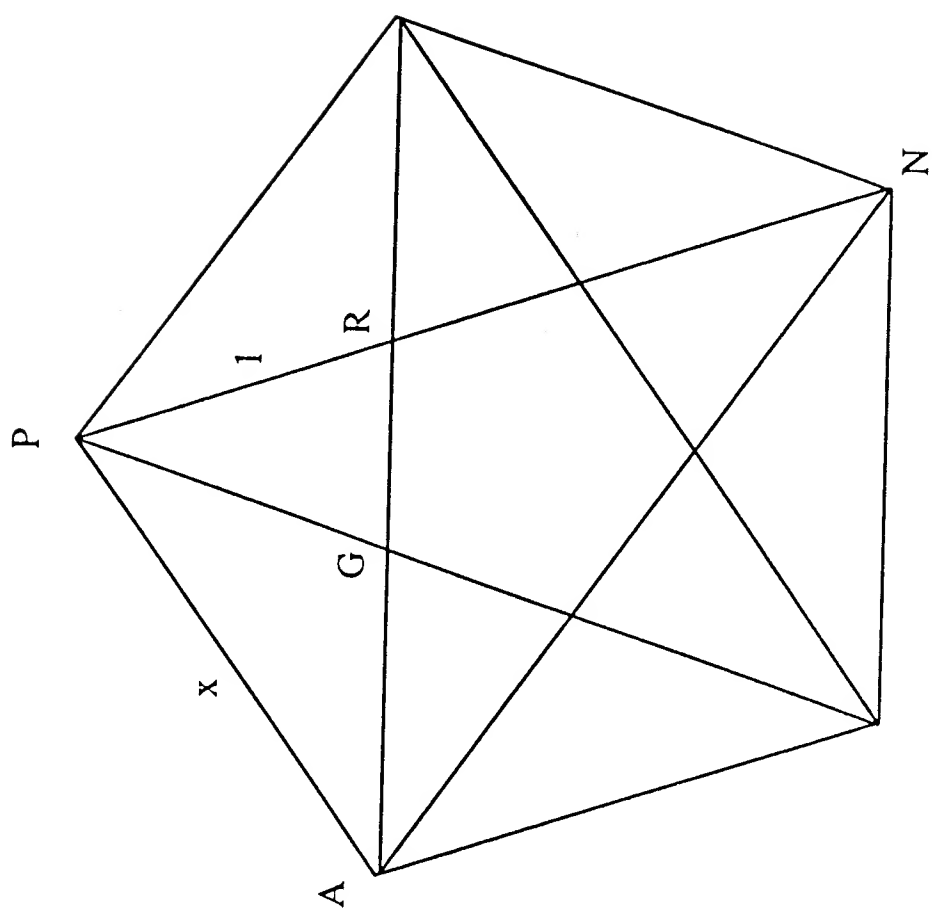
How many such Platonic solids are possible? It is an interesting fact of geometry that there are *only five* possible platonic solids. Apart from the cube and the tetrahedron, there is the **octohedron**, which has eight faces, all equilateral triangles, the **dodecahedron**, which has 12 faces, all pentagons and the **icosohedron** which has 20 faces, all equilateral triangles. This was known to the Greek philosopher Plato, who lived over 2000 years ago, and after whom the solids are named. Plato thought that the Platonic solids held the key to the structure of the heavens, something which the astronomer and physicist Kepler, who lived in the 17th century also believed for a while.

To make the Platonic solids, take straws of equal lengths and join them in the shape of the Platonic solid. The joints are made from rounded paper clips which can be bent to any desired angle at the junction. The two 'U' shaped arms of the clip are pushed into the opening of straws. Open up these arms a little for a better fit.

All these Platonic solids, if hollow, can be fitted exactly or nested one within the other. Some of the nesting combinations are interesting since the corners of the inner Platonic solid touch the vertices or the edges of the outer solid. One such sequence of combinations, with all the five platonic solids nested one within the other is interesting.

In this sequence the outer most platonic solid is the Dodecahedron. A cube can be nested within the dodecahedron. The twelve faces of the Dodecahedron are regular pentagons. On one of the faces join two opposite corners. This forms one edge of a **cube** which can be nested inside dodecahedron. The edge-length of the dodecahedron is 0.618 times the edge-length of the cube. These two nested solids can be made again with straws and paper clips.

By joining one diagonal on each face of the cube, one obtains a tetrahedron. The



length of the edge of the tetrahedron is then  $\sqrt{2}$  times the length of the edge of the cube. This nested combination can also be got from straws and paper clips.

Joining the mid-points of all the edges of the tetrahedron gives an octahedron. The edge of the octahedron is then half the edge of the tetrahedron in length. To make this combination, first make the tetrahedron and mark the midpoints on all the straws. Take pieces of straw which are half the length of the straws on the tetrahedron (a little less than half actually). Now pass a needle and thread through the midpoint of a side of the tetrahedron, slip in the smaller piece through the thread and pass the needle and thread through the midpoint of the adjacent edge of the tetrahedron. Making the thread taut, we find that one edge of the octahedron is in place. Similarly join all the edges of the octahedron with needle and thread.

Inside the octahedron the remaining platonic solid, the icosahedron can be fitted. Divide each edge of the octahedron in the golden ratio 1:1.618 and mark the points. Join the points on adjacent sides in cyclic fashion and you get an **Icosahedron**. To make this combination with straws, first make the octahedron with straws and paper clip joints. Now take pieces of straw which are  $\frac{1}{1.618}$  times the length of the straw forming one edge of the octahedron. (Actually the pieces need to be somewhat smaller so that they fit in nicely.) Mark the point on all the edges of the octahedron which divide the edges in the ratio 1:1.618. On adjacent sides these points must not be equidistant from the vertex, but must alternate in the ratio 1 and 1.618. Pass a needle and thread through these points, thread in the smaller pieces and make the icosahedron. This will require some patience, but the resulting figure will be worth the effort.

## 11 Pentagrams and the Golden ratio

*Materials required: thin strip of paper about 2 inches wide, plain paper and pen.*

A pentagram is a regular pentagon with all possible lines joining corners drawn inside it, creating a starlike effect. The pentagram has such beautiful symmetry that it has been a powerful symbol in many cultures throughout history.

A pentagram is shown in the figure alongside. Ask students to try and answer the following questions by studying the figure.

1. Consider  $\triangle APR$  in the figure. Which other triangles is it similar to?
2. How many triangles can you spot which are congruent to  $\triangle APR$ ?

3. Can you find all triangles congruent to those mentioned in item 1?
4. Can you find the value of  $x$ ? (Hint: use the similarity relation.) Express  $x$  both in the radical form and in the decimal form.
5. Compute the value of  $1/x$  and  $x^2$ . What do you notice? Can you arrive at this relation from the expression for  $x$  in the radical form?
6. Write a few terms of the Fibonacci series. The series starts with 1. The second term of the series is also 1. To get subsequent terms we add the preceding two terms. The third term hence is 2 and the fourth term is 3. Now, assume that the ratio of successive terms in the series converges to a certain value as the number of terms become large. Can you find this ratio? Use the fact that each term is a sum of the preceding two terms.

It is not possible to construct the regular pentagon with ruler and compass. However there is a simple way in which a regular pentagon and hence the pentagram can be obtained. Take a 2 inch wide strip of paper which is about a foot long. Tie it into a simple knot taking care to see that the paper strip retains its full width inside the knot. Keeping it flat in this way and tying the knot carefully somewhat like a knot in a necktie, one obtains a pentagon. Draw all the diagonals of the pentagon. The resulting figure is a pentagram.

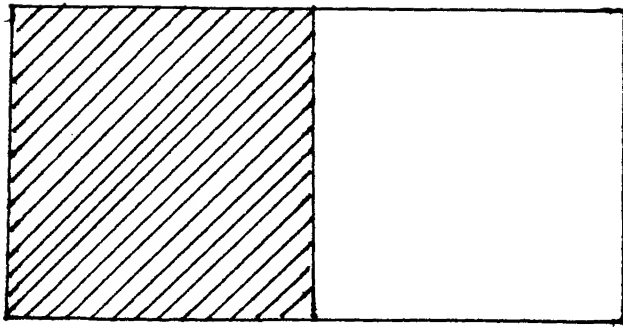
The golden ratio appears many times inside the pentagram. In the pentagram shown here, it is the ratio  $x : 1$ .  $x$  and 1 are the lengths of two unequal sides of the isosceles  $\triangle PAR$ .

If you observe carefully, you will find that there are ten triangles in the pentagram which are the same size and shape as  $\triangle PAR$ .  $\triangle PGR$  is smaller, but it is similar to  $\triangle PAR$ . There are five of these small triangles. Finally there is the big  $\triangle PAN$ , which is also similar to  $\triangle PAR$ . There are five of these big triangles.

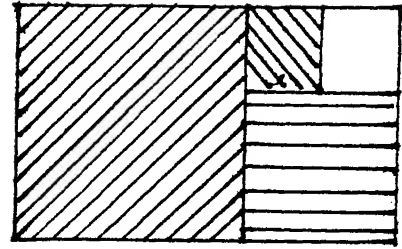
In all these isosceles triangles the lengths of unequal sides bear the golden ratio. From this property the value of the golden ratio can be derived. An approximate value for the golden ratio is 1.618.

One of the peculiar properties of the golden ratio is that its reciprocal is equal to one minus the golden ratio ( $\frac{1}{x} = 1 - x$ ). Another peculiar property is that the square of the golden ratio is equal to one plus the golden ratio.

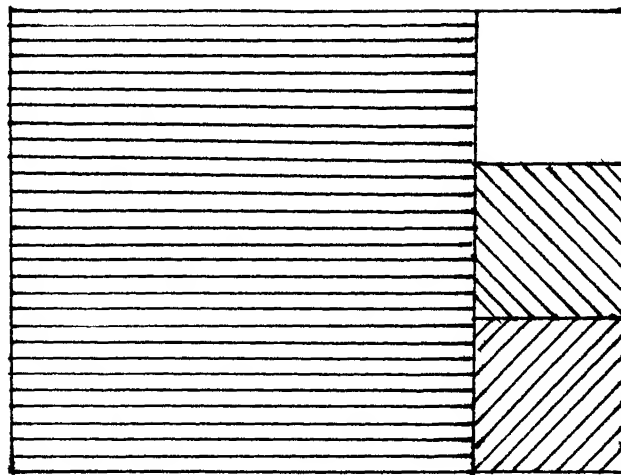
A golden rectangle is a rectangle which has its sides in the golden ratio. When three of these rectangles intersect each other at right angles and their mid-points coincide, you can join all the corners and get a regular **Icosahedron**.



GCD OF 8, 4 is 4



GCD OF 5, 3 is 1.



GCD OF 8, 6 is 2.

---

GCD BY GEOMETRIC WAY.

The ratio between two successive numbers in the Fibonacci series converges to the value of the golden ratio.

The golden ratio has many connections with art, architecture, natural phenomena and regular decagons.

## 12 Greatest Common Divisor by geometry

*Materials required: a piece of paper, a scale and a pencil.*

The G.C.D. of a pair of integers can be found by an interesting geometric method. (G.C.D. of a set of integers is the largest integer which can divide all the numbers without remainder.) In order to do this, we only need a piece of paper, a scale and a pencil.

Suppose the two integers whose G.C.D. is to be found are  $a$  and  $b$ . First draw a rectangle on the paper of length  $a$  and breadth  $b$ . (If  $a$  and  $b$  are large take the length and breadth as  $\frac{a}{2}$  and  $\frac{b}{2}$ , or in some suitable proportion.)

From this rectangle mark off the largest possible square. If  $a$  is greater than  $b$ , this will be a square of side  $b$ . After marking off the square, the portion which remains is a rectangle with sides  $b$  and  $a - b$ . Again mark off the largest possible square from this rectangle. Continue this process till you obtain a square instead of a rectangle. The measure of the side of this square is equal to the G.C.D. of the original pair of numbers.

This method is based on the fact that if  $a$  and  $b$  are both divisible by a number, then  $a - b$  will also be divisible by the same number. The same principle is applied recursively to obtain the G.C.D.

## 13 Special geometric curves

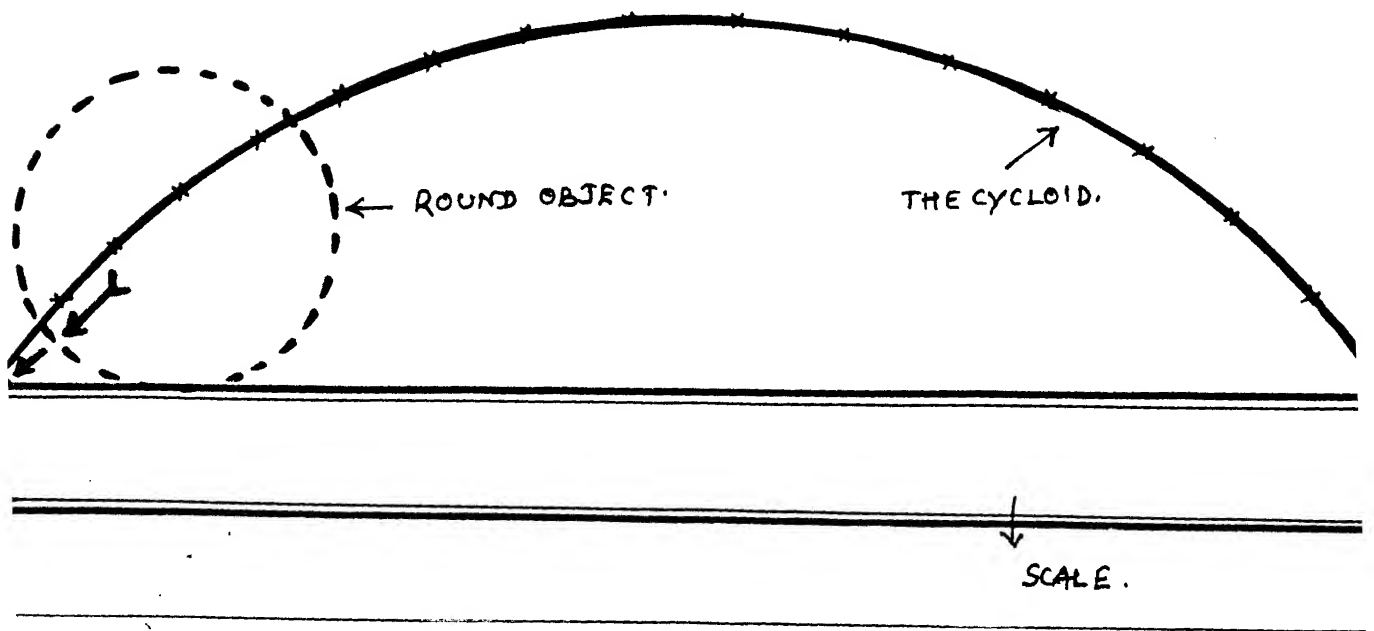
*Materials required: Ruler, carrom striker, pen, mild steel wires for double helix.*

Many types of interesting geometric curves occur in nature. It is possible to draw these curves using simple tools. We have included here techniques for drawing a few of these curves.

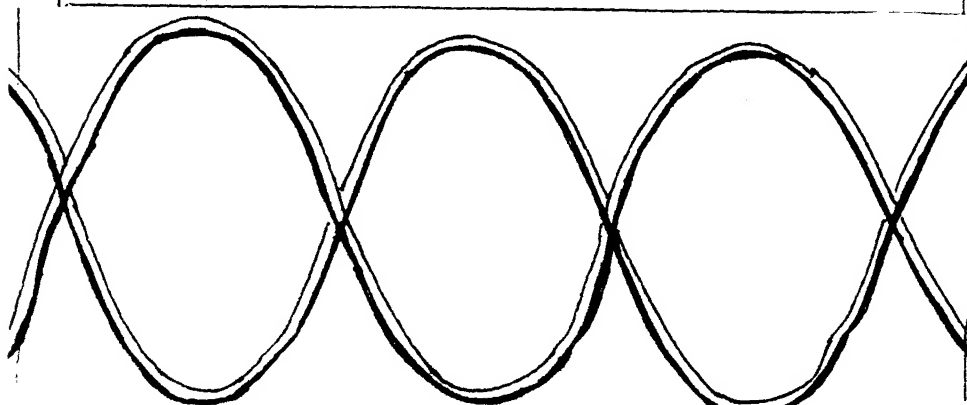
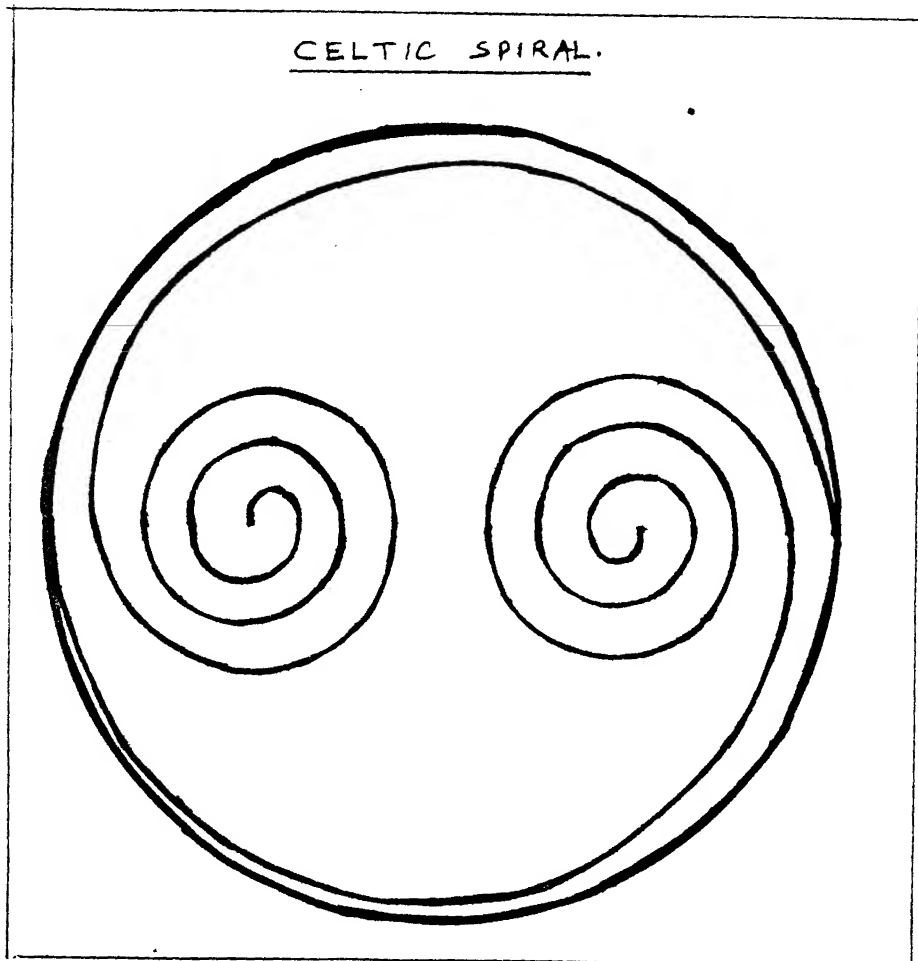
**Cycloid:** To draw this curve, place a ruler on a sheet of paper and fix with cello tape. Draw the edge of the ruler on the paper. On a carrom striker mark a point on the circumference with a pen. Place the striker against the ruler on the paper, so



A CYCLOID



CELTIC SPIRAL.



DOUBLE  
HELIX

that the point marked touches the ruler. Now without sliding the striker, carefully roll it on the edge of the scale till it moves forward by 2 cm. Now mark the point on the paper just below the point marked on the striker. Roll the striker forward again carefully by another 2 cm. Again mark the point on the paper under the point marked on the striker. Repeat this till the striker completes a full cycle and again touches the ruler. Remove the ruler and striker and join all the points marked on the paper by a smooth curve. This curve is a cycloid.

**Celtic spiral:** Celtic spirals were drawn by an ancient people called the Celts who migrated across Europe to decorate their art. To draw a celtic spiral, draw a circle with a radius of 8cm. With a light pencil draw a diameter of the circle. Mark off one cm parts on the diameter, so that there are 16 parts on it. Now with the free hand try and draw the pattern of spirals shown in the figure. Note that both the spirals are clockwise. One starts from below the diameter and another from above the diameter.

**Double helix:** A helix is a three dimensional spiral that looks like the thread on a screw. It is actually a line going at a constant angle around a cylinder or a cone. A common spring is a simple example of a helix. A double helix has two similar helices wound round each other. This structure is one of the most important structures in the real world, for it is the structure of the DNA molecule.

To make a helix, take a tube of cardboard or metal, or a cylinder which is easily available. Take a rectangular piece of paper which exactly rolls on the cylinder. On the rectangular paper, start from one corner (say point A) and draw a line at any angle such that it cuts two longer sides of the rectangle (the sides corresponding to the height of the cylinder) at points A and B. Now mark a point C exactly opposite to point B on the same side as point A. Draw another line parallel to the first line. Continue this process till the lines go out of the rectangle.

Now roll the paper on the cylinder so that the lines are visible. The lines form a helix on the cylinder. Roll a piece of wire made of mild steel (which is commonly available in hardware shops) on the line and press it down so that it takes the shape of a helix.

To make a double helix make two identical helices with wire. These can be mounted on a wooden board to represent a DNA molecule. Small balls made of modelling clay can be placed on the two helices to represent the DNA bases.

1 UNIT											
$\frac{1}{2}$						$\frac{2}{2}$					
$\frac{1}{3}$				$\frac{2}{3}$				$\frac{3}{3}$			
$\frac{1}{4}$			$\frac{2}{4}$			$\frac{3}{4}$			$\frac{4}{4}$		
$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$			
$\frac{1}{6}$		$\frac{2}{6}$		$\frac{3}{6}$		$\frac{4}{6}$		$\frac{5}{6}$		$\frac{6}{6}$	
$\frac{1}{7}$		$\frac{2}{7}$		$\frac{3}{7}$		$\frac{4}{7}$		$\frac{5}{7}$		$\frac{6}{7}$	
$\frac{1}{8}$		$\frac{2}{8}$		$\frac{3}{8}$		$\frac{4}{8}$		$\frac{5}{8}$		$\frac{6}{8}$	
$\frac{1}{9}$		$\frac{2}{9}$		$\frac{3}{9}$		$\frac{4}{9}$		$\frac{5}{9}$		$\frac{6}{9}$	
$\frac{1}{10}$		$\frac{2}{10}$		$\frac{3}{10}$		$\frac{4}{10}$		$\frac{5}{10}$		$\frac{6}{10}$	
$\frac{1}{11}$		$\frac{2}{11}$		$\frac{3}{11}$		$\frac{4}{11}$		$\frac{5}{11}$		$\frac{6}{11}$	
$\frac{1}{12}$		$\frac{2}{12}$		$\frac{3}{12}$		$\frac{4}{12}$		$\frac{5}{12}$		$\frac{6}{12}$	

FRACTION CHART.

## 14 Soap Films

*Materials required:*

Soap films provide fascinating material for mathematical study. It is possible to obtain very interesting and perfectly shaped three dimensional figures using soap solutions which otherwise are difficult to create or to imagine.

To make the soap solution, pour two handfuls of blue detergent powder into half a bucket of water. If the water is hard in your area, then you need to use distilled water. Add about 25-30 ml of glycerine which strengthens the soap films obtained.

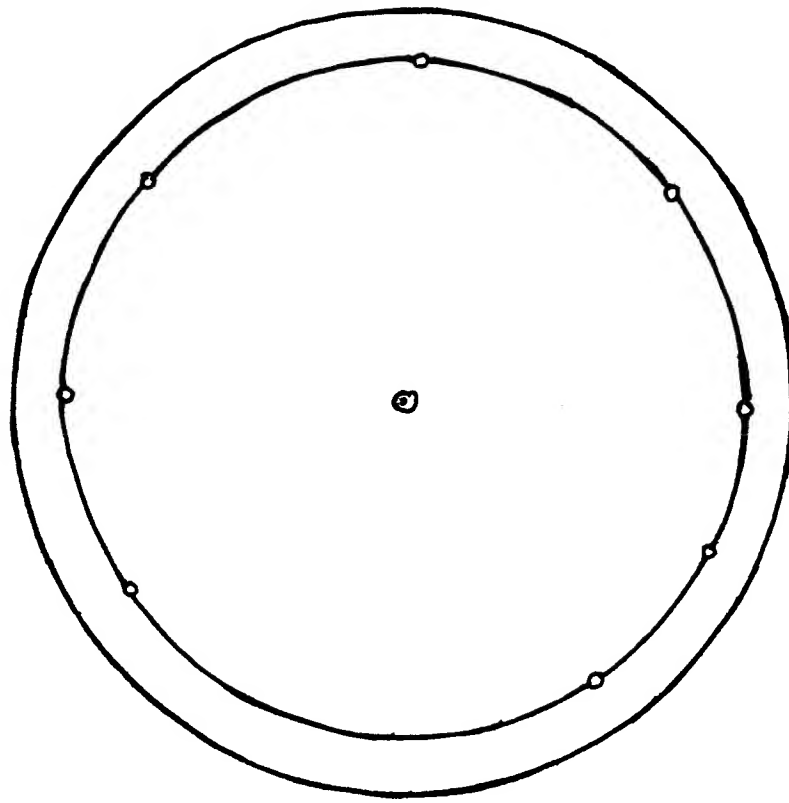
Now make wire frames using mild steel wire in different shapes. If two circular loops held one above the other are dipped in the soap solution a film stretches across both of them. By pulling the loops apart, we obtain a three dimensional shape called a catenoid. A cubical frame dipped in soap solution and pulled out carefully reveals a beautiful pattern of surfaces. There is a small cube of soap film present in the centre which is connected to the edges by planes. The figure is a projection of a 4-dimensional hypercube in three dimensions. A frame which is shaped like tetrahedron dipped in the solution similarly shows a small tetrahedron of soap film in the center. However the tetrahedron in the center has beautiful curved surfaces and edges. When bubble is placed between two parallel transparent plates (made of acrylic or glass), the bubble transforms into a perfect cylinder. It is quite interesting to examine what happens when more bubbles are placed between the plates.

## 15 Fraction chart (A teaching aid)

*Materials required: Plywood or cardboard, Chart paper, string, and bob*

The concept of fractions of a whole is introduced in primary school. Students find it difficult to master and often even understand the concept of a fraction and the meaning of the numerator and denominator. A fraction chart is a very useful teaching aid which can be used for this purpose.

A fraction chart is made from a piece of plywood or thick cardboard which is large enough to be put up on the wall. Narrow strips of chart paper of equal length are pasted on the board at equal distances. Let the first strip represent 1. Divide the next strip into two equal halves and mark the fractions  $\frac{1}{2}$  and  $\frac{2}{2}$ . Divide the next strip into three equal parts and mark the fractions  $\frac{1}{3}$ ,  $\frac{2}{3}$  and  $\frac{3}{3}$ . Continue in this way till all the strips are divided to obtain smaller and smaller fractions. One



A TEACHING AID TO STUDY THE PROPERTY OF CIRCLE .

can make a chart till the fractions  $\frac{1}{20}, \frac{2}{20} \dots$  if space is available. Now suspend two long strings from the top of the board with bobs attached at the end. The strings remains vertical like a plumbline.

The fraction chart can be used for showing the part whole relationship: how many one-thirds make up one? Another important use of the chart is to show equivalent fractions. Drop the plumbline over a fraction, and if the chart is aligned vertically all the fractions which coincide with the plumbline are equivalent fractions. The students also learn that any fraction of the form  $\frac{n}{n}$  is equal to 1. It is also possible to do some rudimentary addition and subtraction of fractions with the chart. If two fractions are to be added find their equivalent fractions on the same line of the chart by dropping the plumbline. Now it is possible to add the fractions easily by adding the numerators.

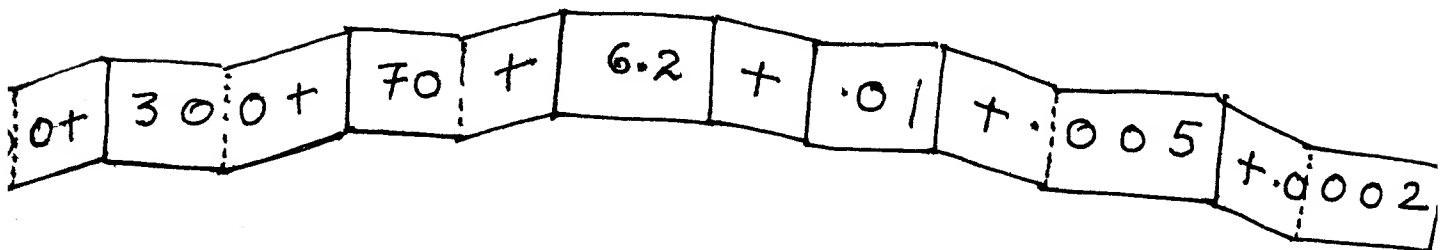
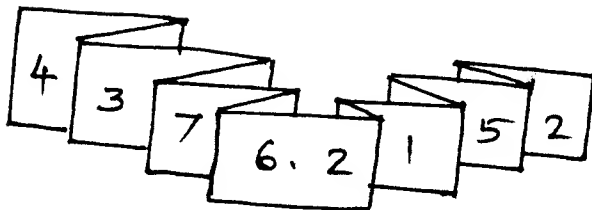
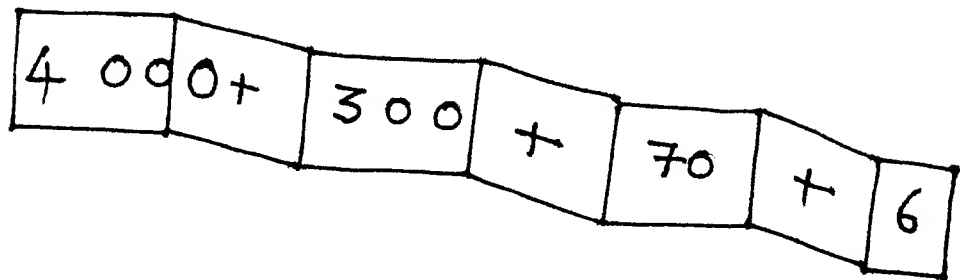
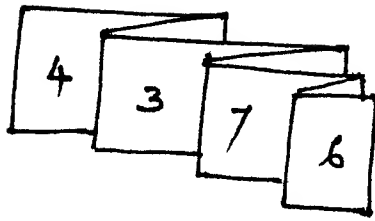
## 16 Property of circles (A teaching aid)

*Materials required: Plywood or acrylic board cut in the form of a circle, pegs, rubber bands*

This is a teaching aid which can be used to illustrate the angle properties of a circle. These properties are generally not obvious to most students.

The teaching aid consists of a circular piece of plywood or acrylic of about 8 inch diameter. An inner circle of about 7 inch diameter is marked out on the acrylic or wood. A few holes are drilled on the inner circle and at the centre as shown in the figure. At least some of the holes on the circle must be diametrically opposite to each other.

Put pegs on two holes which are diametrically opposite to each other and stretch a rubberband over the two pegs. With the third peg on any other hole, measure the angle subtended by the diameter on the circumference. By stretching a rubber band over four pegs on the circumference verify the angle property of a cyclic quadrilateral. Similarly verify the equality of angles in the same segment. The alternate segment theorem can also be demonstrated with this teaching aid.



## 17 Place value snake

*Materials required: Strip of paper, pen*

The concept of place value is not easy to grasp for very young children. The place-value snake is a delightful activity which helps in fixing the concept in the mind of a young child.

To make a place value snake, take a strip of paper and fold it in the manner shown in the figure. Write a number, say 4376, on the visible part of the folds as seen in the figure. Hidden away in the folds of the paper are zeroes and '+'s. When the snake is opened, the expanded form of the number appears.

Any child can make the place value snake and show it to his friends. In the course of doing this he learns to associate the place of a digit with a power of 10. The snake can also include decimal numbers as shown in the figure.



$\frac{\sqrt{3}}{2}$ $\tan 60^\circ$	$\frac{\sqrt{2}}{2}$ $\sec 45^\circ$	1 $\sin 45^\circ$	1 $\sin 90^\circ$	$\frac{\sqrt{2}}{2}$ $\cot 45^\circ$	0
0 $\sec 30^\circ$	2 $\csc 30^\circ$	0 $\sin 0^\circ$	0 $\tan 45^\circ$	$\frac{\sqrt{3}}{2}$ $\sin 60^\circ$	$\frac{\sqrt{3}}{2}$ $\sin 60^\circ$
$\frac{1}{2}$ $\cot 60^\circ$	0 $\cot 0^\circ$	1 $\sec 60^\circ$	0 $\tan 30^\circ$	1 $\cot 45^\circ$	$\frac{\sqrt{3}}{2}$ $\cot 45^\circ$
1 $\sin 30^\circ$	$\frac{1}{2}$ $\sin 30^\circ$	0 $\cot 0^\circ$	0 $\cot 90^\circ$	1 $\cot 90^\circ$	$\frac{\sqrt{2}}{2}$ $\cot 45^\circ$
2 $\csc 30^\circ$	1 $\csc 45^\circ$	0 $\csc 45^\circ$	0 $\csc 90^\circ$	1 $\csc 90^\circ$	$\frac{\sqrt{2}}{2}$ $\csc 45^\circ$
$\frac{\sqrt{3}}{2}$ $\tan 60^\circ$	$\frac{\sqrt{2}}{2}$ $\sec 45^\circ$	1 $\sin 45^\circ$	1 $\sin 90^\circ$	$\frac{\sqrt{2}}{2}$ $\cot 45^\circ$	0

# DESIGN FOR MAKING TRIGONOMETRY PUZZLE CARDS.

$\frac{\tan A}{\sec A}$ $\sin^2 A$ $\frac{1 - \sin A}{1 + \sin A}$	$\cot A$ $\tan A$ $\frac{\cos^2 A}{1 + \sin A}$	$\cot A \sin A$ $1 - \cos A$ $\frac{1 + \cos A}{1 - \cos A}$	$\sin^2 A + \cos^2 A$ $\sec^2 A - \tan^2 A$ $\tan A$
$\cot A$ $\tan^2 A$ $\frac{1 - \tan^2 A}{1 + \tan^2 A}$	$\sec A$ $\frac{1}{\cos A}$ $\frac{1}{\sec A}$	$\sec A$ $\frac{1}{\cos A}$ $\frac{1}{\sec A}$	$\tan A$ $\frac{1}{\sec A}$ $\frac{1}{\sec^2 A}$
$\sec A$ $\frac{1}{\cos A}$ $\frac{1}{\sec A}$	$\tan A \cos A$ $\cos A \sec A$ $1$	$\sec A$ $\frac{1}{\cos A}$ $\frac{1}{\sec A}$	$\cot^2 A + 1$ $\csc^2 A - \cot^2 A$ $\csc A$
$\sin A$ $\frac{1 + \cos A}{2}$ $\frac{1 - \cos A}{2}$	$\sin A \cos A$ $\cos A \sec A$ $1$	$\sin A$ $\frac{1 + \cos A}{2}$ $\frac{1 - \cos A}{2}$	$\sin A$ $\frac{1 + \cos A}{2}$ $\frac{1 - \cos A}{2}$
$\cos^2 A$ $\frac{1}{\sec^2 A}$ $\frac{1}{\sec A}$	$\sin^2 A$ $\frac{1}{\sec^2 A}$ $\frac{1}{\sec A}$	$\sin^2 A$ $\frac{1}{\sec^2 A}$ $\frac{1}{\sec A}$	$\sin^2 A$ $\frac{1}{\sec^2 A}$ $\frac{1}{\sec A}$

## PART II

### Puzzles and Games

#### 18 Trigonometry puzzle

*Materials required: Strip of paper, pen*

This is a puzzle similar to a jigsaw puzzle, where students have to put together 16 square pieces to form a larger square. All the pieces have trigonometric ratios or numerical values written along their four edges. The pieces will have to be assembled in such a way that at each joint, the trigonometric ratio and the numerical value correspond. Students are easily attracted to the game and can spend a long time attempting to solve the puzzle. A sample chart of 8in x 8in is shown in the figure, from which the 16 pieces can be cut. The level of difficulty can be reduced by marking the outside edges and the corners differently instead of with numerical values.

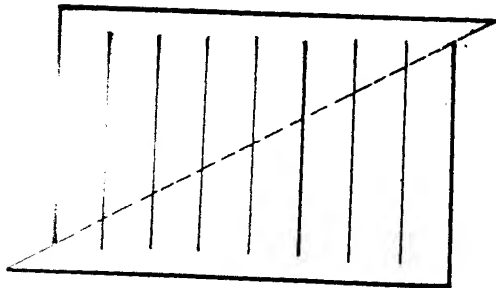
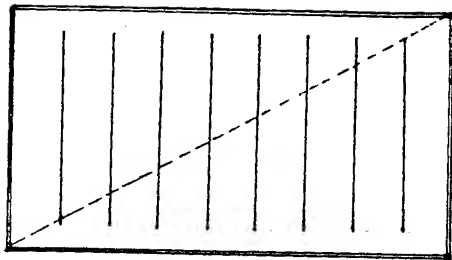
Another chart with trigonometric identities is also shown. After the first game becomes easy, the second chart game can be introduced. It is also possible to design similar charts which can be cut into pieces shaped like triangles or hexagons.

#### 19 Missing squares and lines

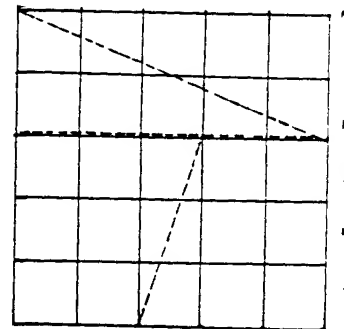
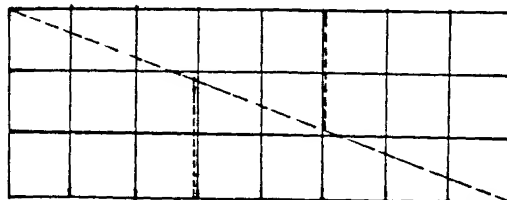
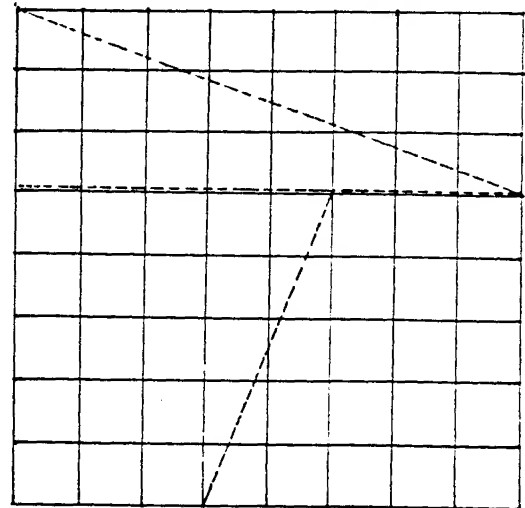
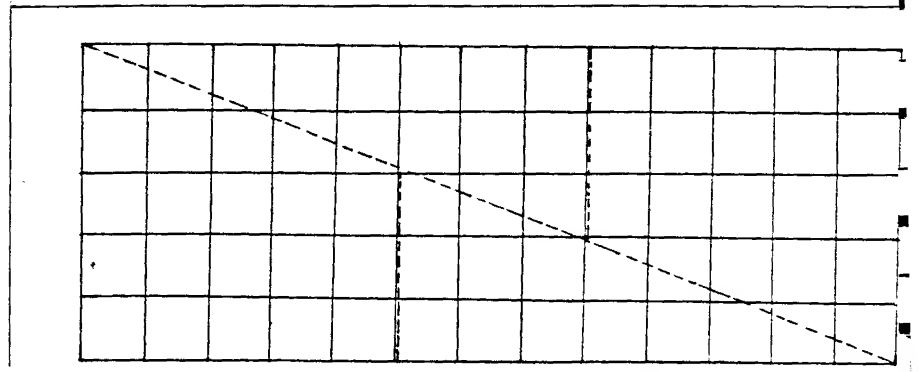
*Materials required: Card paper, graph paper(optional), pencil or pen, cutter or scissors.*

The missing square and missing line activities take everyone by surprise. But there is a perfectly clear explanation for the mysterious missing square and missing line.

**The missing square:** To make the missing square puzzle, cut a paper into 5" by 5" square. Form 25 smaller squares by drawing lines for every inch. Now cut this square along the dotted line shown in the figure to get four pieces. These four pieces on rearranging into a rectangle give us only  $8 \times 3 = 24$  squares, leaving us surprised as one square is missing from the original piece. The same activity can be performed on a 8" x 8" square and cutting it into four pieces as above. On rearranging the pieces into a rectangle we get  $13 \times 5 = 65$  squares, this time one more than the original.



MISSING LINE SEGMENT



MISSING SQUARES

The reason for this increase or decrease in the number of square is simple. In each case when we cut the square into pieces and re-assemble them to form the rectangle, the small squares that are obtained are not perfect squares, but are slightly enlarged or compressed. Sometimes the edges do not match. In order to obtain perfect squares, the pieces have to overlap slightly. The total area of overlap will be equal to the area of one square, thus bringing down the number of squares from say 25 to 24. Alternatively gaps may have to be left along the junction lines to get perfect squares. In this case area is added and the total increase in area is equal to the area of one square. The number of squares then increases say from 64 to 65.

**The missing line:** If a rectangle as shown in the figure is cut along a dotted line and shifted as shown, one of the original eight line segments inside the rectangle disappears. What happened to the missing line? Tell the students to observe what is happening while sliding the two halves. When we slide the two pieces, one against the other, one line segment in each half becomes the boundary of the rectangle. And the lengths of the remaining line segments are slightly increased, the total length of the eight segments remains the same.

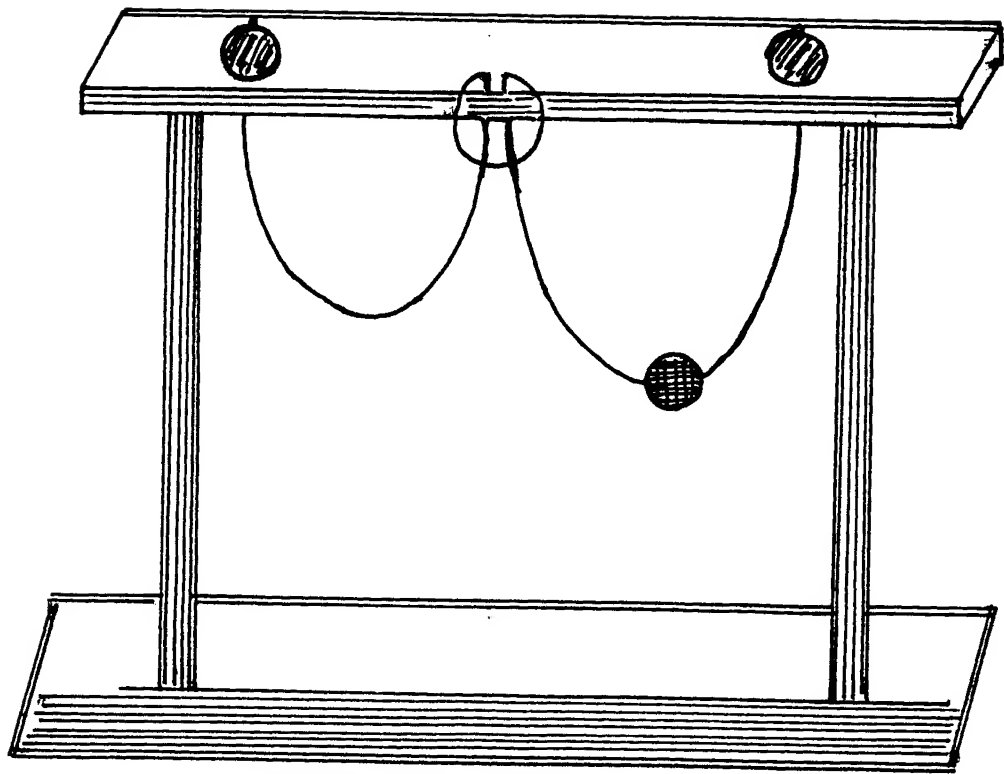
## 20 Topology games and puzzles

In this section we have included games based on elementary principles of topology. Most of these games also contain an element of surprise and are entertaining.

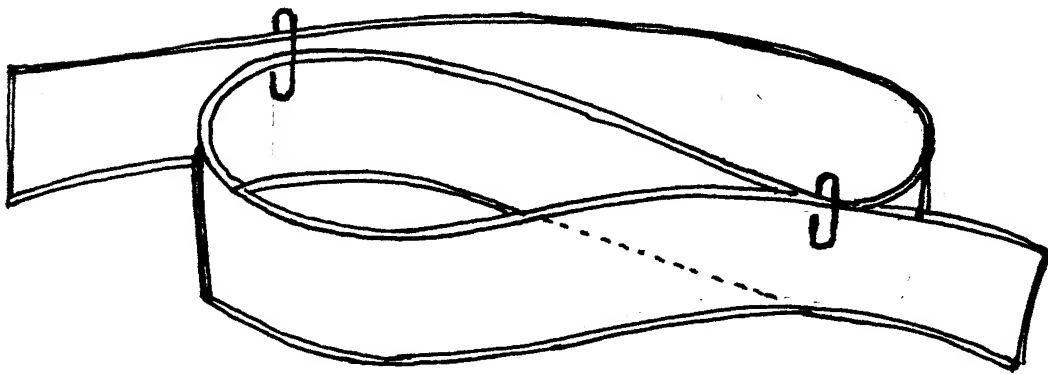
### Button hole puzzle:

*Materials required: a shirt or a piece of cloth with a button hole, some string, a pencil and cello tape.*

First thread the string through the button hole as shown in the figure. Attach the pencil to the ends of the string with the tape. The puzzle is now to remove the pencil from the button hole without cutting the string. The solution is fairly simple: the pencil needs to be pulled through the loop in the string and through the button hole.



STRING AND BEAD



LOCKING PAPER CLIPS

## **Belt and buckle puzzle:**

*Materials required: belt with a buckle.*

Like the previous puzzle, this is also fairly simple, but may still be new to most students. Many students are familiar with a peculiar change which sometimes takes place in the buckle of an ordinary belt. The pin suddenly is on the wrong side. Children often have no clue about how this happened, or how to get the pin on the right side again, without bending it. This can be posed as a puzzle. The solution lies in taking the belt completely through the buckle once.

## **Locking paper clips:**

*Materials required: strip of paper or transparent plastic, two paper clips.*

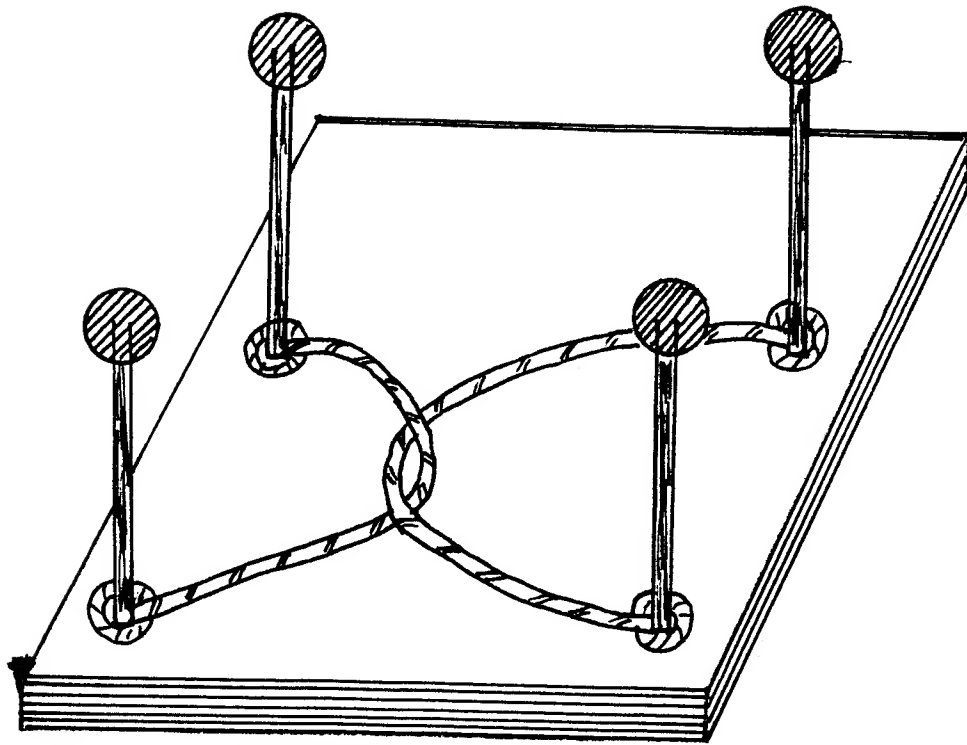
Take a thin strip of paper about one inch wide, preferably cut from OHP transparency film. Fold it at one end to make a loop and secure it with a paper clip. Fold the other end to make a second loop and secure it with another paper clip. The two loops must make a 'figure of 8' as shown in the figure. Now hold the two ends of the paper strip and pull them apart sharply so that the paper strip straightens out. Watch what happens to the paper clips.

## **Interlocked loops :**

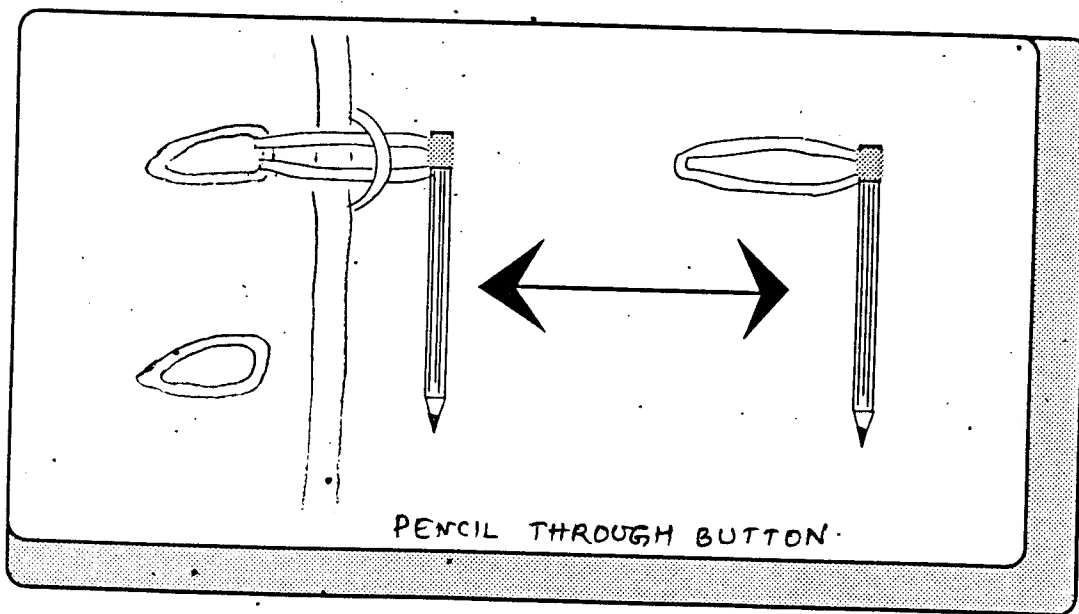
*Materials required: Two pieces of string each about 3 feet long.*

Tie the first of the two strings to both the wrists of a student. Let the loop which passes over the wrists be loose. Tie the second string in a similar manner to the wrists of another student, but first allow it to pass under the first string, so that the two strings are interlocked as shown in the figure. The puzzle is to disentangle the two strings without cutting them.

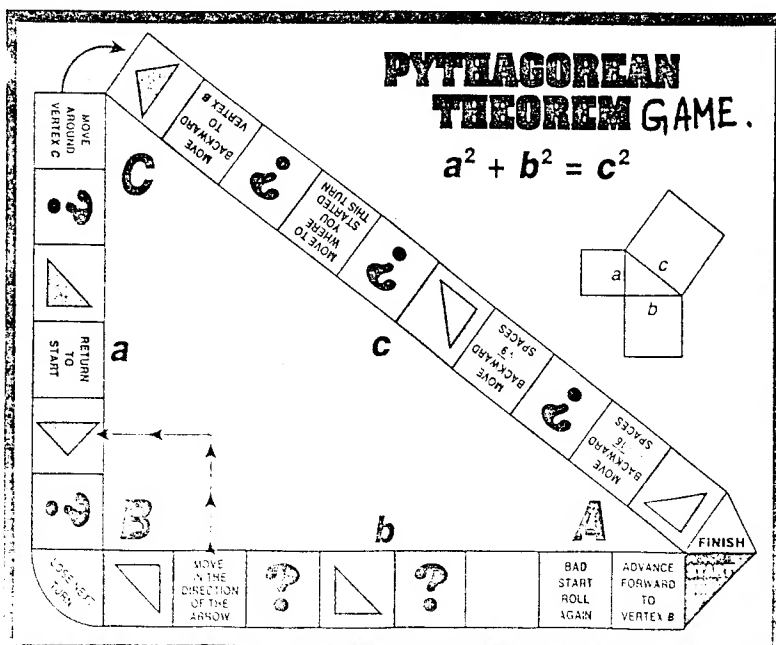
This puzzle is based on the fact that the two strings which look interlocked are not really so. The strings are open at the wrist although the loop appears to be closed here. Imagine that the hand is cut just below where the first string is tied to the wrist. Then it will be easy to take out the second string. Now imagine that the cut moves further and further up the hand. At one point it crosses the loop. Now let the second string which must pass through a cut in the hand also cross the loop. For this you must let it pass through the loop. Now take the second string



ENTANGLED LOOPS.



PENCIL THROUGH BUTTON.



over the fingers (where the hand is actually cut, that is, where the hand ends) and the strings disentangle easily.

### String and bead:

*Materials required:* A long piece of string, three beads and a rectangular piece of wood.

Make three holes of about 1 centimetre diameter in the piece of wood, one in the centre and one near each edge. The beads must be big enough not to pass through the hole. Tie a bead at one end of the string and pass the other end of the string through a edge-hole in the piece of wood. Now take the string through the centre hole so that it forms a loop as shown in the figure. Thread the second bead through the string, take the string out through the other edge-hole in the piece of wood, and tie the last bead to the end of the string. The final arrangement is as shown in the figure. The puzzle is now to take the middle bead to the other loop in the string.

The principle is the same as in the previous puzzle. Hold the loop in the centre of the piece of wood with the thumb and fore-finger, pull it towards one of the edge-holes. Pass it through the edge-hole from below, take it over the edge bead and pull it back through the edge-hole. It becomes partly disentangled. Repeat this at the other edge-hole, and the string comes fully out of the centre hole, forming one large loop. Now move the middle bead to the place where you desire it to be and do the steps in inverse fashion. At the end of the two inverse steps, you are back to the original arrangement, but with the bead now in the other loop.

## 21 Pythagoras theorem game

*Materials required:* Chartpaper for making the board and question cards, cutter, buttons, a pair of dice

This game is meant to familiarize students with the Pythagoras theorem which plays a crucial role in school geometry.

In this game two or more students roll a pair of dice to move around the triangular board shown in the figure. Copy out the board on to chart paper. Make question cards by copying each question on to a separate card.

Both the dice have to be rolled together. Add the squares of the numbers on both the dice and take the square root (from the famous theorem  $a^2 + b^2 = c^2$ ).





Round off the answer to the nearest whole number and move that many spaces.

For example, if you have rolled 1 and 2 on the dice, then  $c^2 = 1^2 + 2^2 = 5$ . Since  $\sqrt{5} = 2.236$  approx., you move 2 spaces.

Follow the instructions given on the square you come to. If your square is marked with a '?', you have to pick a card at random and answer the question on it. If you answer correctly you get a chance to roll a *single* dice and move the number of spaces it shows. A wrong answer means you stay on the square till your next turn.

The first player who goes twice around the board wins.

## 22 Magic mind reader - 1

*Materials required: Chartpaper, pen, cutter.*

Here is a simple game to boggle your mind. Cut out seven cards from chart paper in equal sized rectangles (about 8" x 6"). On the main card copy out the numbers 1 to 20 as shown in the figure. On the remaining 6 cards, cut windows in the pattern shown in the figure and also copy down the numbers on the bottom two rows of each card. Now the game is ready.

Ask your friend to think of a number from the main card. Let him tell you on which of the other six cards (mask cards) the number appears. There will be three cards on which it appears. Stack these three cards together and place it on the main card aligning the edges.

The number your friend thought of appears in the magic window!

How does it work?

There are six mask cards. Any three of them contain a given number. The different ways in which you can select three cards out of six is given by the formula:

$${}_6C_3 = \frac{6!}{(6-3)!3!} = \frac{6 \times 5 \times 4}{3 \times 2} = 20$$

There are twenty different ways of selecting 3 cards out of six. So you can have twenty different numbers shown within the magic window which shifts with each combination of 3 cards.

Look in the main card and you will find that there are twenty numbers, exactly  ${}_6C_3$ .

The windows have to be cut on each card so that each combination of 3 cards gives a unique window. This can be done by copying the pattern in the figure or by trial and error.

<div>1</div> <div>3 5 7 9 11 13</div> <div>15 17 19 21 23 25 27</div> <div>29 31 33 35 37 39 41</div> <div>43 45 47 49</div>	<div>4</div> <div>5 6 7 14 15</div> <div>17 24 25 26 27 34</div> <div>36 37 44 45 46 47</div>
<div>2</div> <div>3 6 7 12 13 16</div> <div>17 22 23 26 27 32 33</div> <div>36 37 42 43 46 47</div>	<div>8</div> <div>9 18 19 28 29 31</div> <div>39 48 49</div>
<div>10</div> <div>11 12 13 14 15</div> <div>16 17 18 19</div>	<div>20</div> <div>21 22 23 24 25</div> <div>26 27 28 29</div>
<div>30</div> <div>31 32 33 34 35</div> <div>36 37 38 39</div>	<div>40</div> <div>41 42 43 44 45</div> <div>46 47 48 49</div>

## 23 Mind reader - 2

*Materials required: Plain paper and pen.*

This is yet another game which looks magical, but has an underlying principle that can be easily grasped. Copy out the tables shown in the figure. That's all that is needed for this game.

Now ask your friend to think of a number less than fifty. Let him say which of the tables shown in the figure contain the number. Add all the corner numbers in the tables which your friend points out. Their sum gives the number your friend thought of!

The principle is so simple that you can make your own tables which are different from these.

To do this first make magic tables for numbers from 1 to 9. You need 4 tables with the corner numbers  $\boxed{1}$ ,  $\boxed{2}$ ,  $\boxed{4}$  and  $\boxed{8}$ . (The box around  $\boxed{1}$  means it is a corner number.) You have to put all the numbers from 1 to 9 in these four tables in such a way that adding the corner numbers of the tables in which a number appears gives that number. In other words, you have to write down all the numbers from 1 to 9 as the sum of some or all of the numbers 1, 2, 4 and 8. 1 and 2 already appear in the tables. 3 can be written as  $1 + 2$ . So it must appear in the tables with  $\boxed{1}$  and  $\boxed{2}$ . 4 is already present. 5 can be written as  $4 + 1$ . Hence it must appear in the tables with  $\boxed{1}$  and  $\boxed{4}$ . Similarly continue for all the numbers upto 9.

The number 10 can be put as a corner number in a new table. Now all the numbers from 11 to 19 can be expressed as the sum of 10 and a single digit number. 15, for example, can be expressed as  $10 + 5$ . so place 15 in the tables in which  $\boxed{10}$  appears and those in which 5 appears. This will be in the tables with  $\boxed{10}$ ,  $\boxed{4}$  and  $\boxed{1}$ . Continue in this way upto the number 19. For numbers from 20 to 29 make another table and so on upto 49. This is the way in which the tables shown here are constructed.

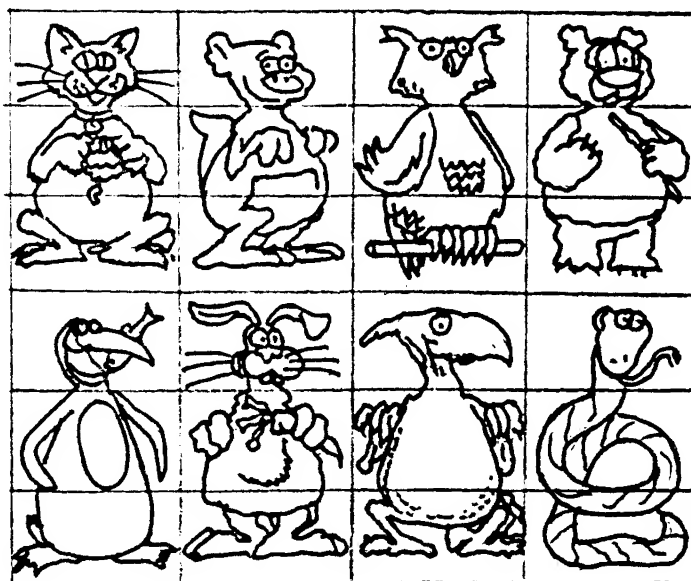
Now try changing the corner numbers to  $\boxed{1}$ ,  $\boxed{2}$ ,  $\boxed{3}$  and  $\boxed{5}$ . Can you generate another set of tables?

The table shown in the figure is not economical. There is no need to start a new table for the number 10. We can express it as  $8 + 2$ . Hence it can be fitted in the tables with  $\boxed{8}$  and  $\boxed{2}$ . In this way how many numbers can you fit within the first 4 tables? Try to make the most economical set of six tables. Find out the greatest number your mind reader is capable of reading with 6 tables. (It should be able to read any number less than or equal to 63.)

$a^2+ab$	$4ab$	$a^2-ab$	$a^2+2ab+b^2$	$-16$	$4a+4b$
$a^2$	$-b^2$	$ab$	$-4$	$-ab$	$b^2$
$ab-b^2$	$-a$	$b$	$-a-b$	$-a^2$	$-4a+4b$
$4a$	$-a^2+ab$	$-a^2-ab$	$4$	$-b^2-ab$	$-4a$
$4a-4b$	$-b$	$-16$	$a^2-b^2$	$-ab+b^2$	$-4b$
$-4a-4b$	$-a+b$	$1$	$a^2-2ab+b^2$	$a$	$b^2+ab$

$a+b$	$a$	$b$	$+4$	$-1$	$-4$	$-b$	$-a$	$a-b$
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TIC-TAC MULTIPLIER.



DESIGN FOR COMBINATION BOOK.

## 24 Tic-tac multiply

*Materials required: Card paper, pen, buttons in two different colours – about 15-20 in each colour.*

This game, which is a modified version of tic-tac-toe (also called dots and crosses) is excellent to sharpen skills of factorization in algebra. It can also be modified to work for younger children and can sharpen single digit or two digit addition and multiplication.

To play the game you need to copy out the board shown here. You also need about 15-20 buttons in two different colours, say red and blue. Each set is given to one player.

One red and one blue button are needed for placing on the factor board below the main board, one for each player. The remaining red and blue buttons are for placing on the main game-board.

The game begins with the players placing both the factor board buttons on any two squares on the factor board. The first player to move will move his factor board button (say red) to any of the factor board squares. Now the first player multiplies the numbers under the two buttons on the factor board and places a new red button on the product on the main board. For example, if on the factor board the blue button is on -1 and the red button on 4. The product is -4. So the first player places a red button on -4 in the main game-board.

Now it is the second player's turn. She too first moves her factor board button, the blue one in this case. She then multiplies the numbers under the two factor board buttons and places a blue button on the product in the main board.

On the factor board the first player can move only the red button, the second player can move only the blue one.

If a player makes a mistake while multiplying, he or she loses a turn. The first person to get 4 small buttons in a line wins.

This game is a constrained version of tic-tac-toe. In ordinary tic-tac-toe, buttons (or dots and crosses) can be placed anywhere on the board. In the constrained version, the factor board limits the number of squares where it is possible to place the button. By varying the type of constraint, one can modify the game to focus on a range of mathematical skills.

## 25 A combination book:

*Materials required: Eight sheets of paper*

This is a picture book containing eight pictures of different animals. Each page is cut into three parts, exactly dividing each picture of an animal into the head, body and feet. The book can be prepared by carefully drawing the pictures given here. The pages can then be stapled together such that the dotted lines in each page are aligned. All the pages can then be cut into three parts along the dotted line.

Now heads, bodies and feet from different animals can be combined to form delightful pictures of strange animals. How many pictures are possible in all? (By keeping a head and a body constant and varying the feet, one can obtain 8 pictures. Now by keeping the head constant and varying the body and the feet, one obtains  $8 \times 8 = 64$  pictures. By also varying the head one obtains a total of 512 pictures.)

## 26 Dominoes

*Materials required: Chart paper, paper clips, pen.*

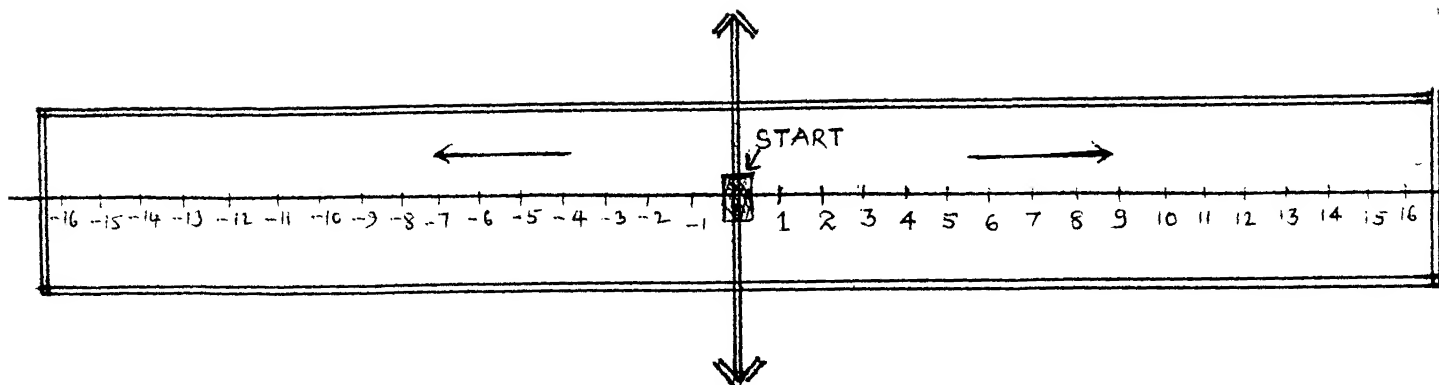
Dominoes are blocks made up of two squares. The activity consists of filling a given square with dominoes. A chessboard is put before the students and they are asked how many dominoes are required to cover the chessboard. Now two opposite corners of the chess board are covered with black paper. How many dominoes are now needed to cover the chessboard? Ask the students to try and cover the chessboard with dominoes which have been cut out beforehand.

A square of  $6 \times 6$  and another of  $4 \times 4$  are available for the students to try out a similar exercise with them. Ask the students to reason out why it is impossible to cover the boards when two opposite squares are blanked out.

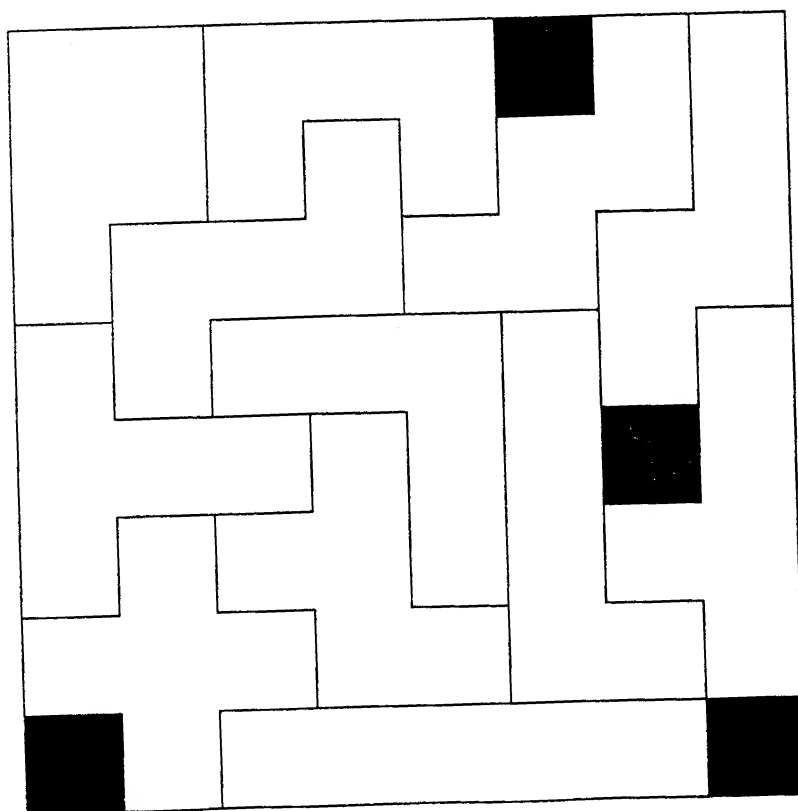
## 27 Pentaminoes

*Materials required: Chart paper, paper clips, pen.*

Pentaminoes are made by joining five squares so that at the joints, one side of a square fully joins with one side of another. There are twelve possible arrangements in which five squares can thus be joined to each other. Hence there are 12 possible pentaminoes.



PLAY BOARD FOR NUMBER LINE GAME.



PENTAMINOS FROM A 8x8 BOARD.



Ask the students to find out all the possible pentaminoes. Let them cut out these pentaminoes from card paper. The total area of the pentaminoes, assuming each small square from which they are made up to be of unit area is  $12 \times 5 = 60$  square units. Give the students a  $8 \times 8$  board which contains 64 squares. Can the square board be covered with all the twelve pentaminoes, so that four squares remain. Although there are many possible ways of doing this, it may take a while to find even one arrangements which fills 60 of the 64 squares. One of these arrangements is shown in the figure. Some of these possible arrangements leave four squares exactly in the middle. Can the students find such solutions?

## 28 Number line checkers

*Materials required: Chart paper or cardboard, pen, buttons in two different colours - 5 buttons in each colour.*

Here's a simple and delightful game which sharpens skills at adding and subtracting signed numbers. This is suitable for children in primary school.

Copy out the number line given in the figure to a convenient size. Two players play the game at a time. They get 5 buttons each in different colours, say red buttons for the first player and blue buttons for the second.

In the beginning of the game all the buttons are placed at the starting point which is zero.

Now the first player picks a card. There is a number written on it. He moves one of the red buttons to this number on the number line. Now it is the second player's turn. She too picks up a card and a blue button on the number line on top of the number which appears in her card.

Everytime a player picks a card he or she can move a fresh red button to the number on the card *or* can also move a button already on the number line by the number of places shown on the card. For example, if there is already a button on -5 and the card shows -7, the player can move the button to -12.

The aim of each player is to capture as many buttons as possible. Both red and blue buttons can be captured by each player.

A player can capture buttons in two ways:

- A player's button moves past -19 or +19. Then he or she win the button.

- One player's button jumps over the second player's button. Then the second player's button is 'killed' and the first player gets to keep it.

The game stops when either all red buttons or all blue buttons have left the number line.

The person with the maximum number of buttons with her or him (both red and blue) wins.

Some additional rules:

- Two buttons cannot occupy the same position on the number line.
- If a player makes a mistake in arithmetic while moving the button, the button goes back to the starting point.
- A player can jump over his or her own button. No problem. The button stays alive.
- If a player jumps over two (or even three) of the other player's buttons, he or she gets to keep all of them!

## List of Sources

Some of the activities, games and puzzles described here are taken from the following sources and modified or developed:

1. The Mathematics Teacher, Vols 84-88, 1992-1996.
2. How Mathematics Works, by Carol Vorderman, Dorling Kindersley, London, 1996.

# **APPENDIX**

## **Annotated Compilation from other sources**

### **SECTION - A, GEOMETRY.**

#### **1. EXPLORING DIFFERENT PROOFS FOR THE PYTHAGORAS THEOREM.**

Pythagoras theorem forms a very important unit of the high school geometry. There are different proofs for this theorem. In this activity students discover six different ways of proving the theorem. There are six different figures given to different groups in the class. The students have to arrive at the proofs through discussion.

This activity helps the students to develop mathematical and communication skills. Also the students come to know about a theorem rich in the history of various methods of proof.

(class 6-10)

Source: The Mathematics teacher, Vol.88, Feb. 1995.

#### **2. LINE AND ROTATIONAL SYMMETRY BY PAPER FOLDING.**

Line and rotational symmetry of geometrical figures are not easy concepts in the high school geometry. These concepts can be explained by paper folding through this activity.

First each student is given a set of paper pieces cut in the form of a square, a triangle, a rectangle, a rhombus and some floral designs. The students then fold these papers along a line such that each half overlaps on the other exactly. In this way they discover the line of symmetry. They are asked to find out all lines of symmetry for a given figure. Further the rotational symmetry for a given figure is also to be found out by rotating the figure for a degree of angle.

An interesting problem to explore for the students is to produce a design with as many lines of symmetry as possible for a geometric figure folded once, twice or thrice.

(class 7-10)

Source: The Mathematics teacher, Vol.84, Apr. 1991

### **3. PAPER FOLDING ACTIVITY FOR COUNTING THE SLOPE.**

In high school coordinate geometry it is taught that the slope of a line and the slope of its perpendicular are negative reciprocals of each other. By performing this activity students can actually discover this relationship.

The students are divided into small groups and each group is given six (the number can be changed) graph sheets each having a pair of line segments drawn on them. First the slope of the line is computed by counting the number of squares vertically and horizontally. Further the paper is folded in such a way that a perpendicular crease is formed to the line segment. Now the slope of this perpendicular line is computed. For this three points can be marked on the crease such that they have integral coordinates and the slope is counted for the perpendicular crease. Thus it can be discovered that the slopes are negative reciprocals of each other.

(class 8-10)

Source: The Mathematics teacher, Vol.83, Sept. 1990.

### **. LEARN GEOMETRY BY ACTIVITY.**

In this section we have a series of activities for the students to perform such that they explore some very important basic geometrical concepts like line segment, angle, triangle, circle.

For performing the activity we require a cork board, push pins, strings and scale. First by using strings and push pins the line segments, angles of different types and a triangle are formed and their properties studied.

In the next activity a circular ring is placed on the cork board and by using the string or rubber band the different parts of the circle explored. Drinking straws are used to form solid figures and their properties are studied.

(class 5-8)

Source: The Mathematics teacher, Vol.87, Nov. 1994.

## 5. ACTIVITY USING FLEXIGONS.

Flexigons are created by stringing together lengths of plastic drinking straws with nylon fishing line and knotting the ends of the curve to form a closed figure. Depending on the number of straws used we can make polygons of different sides.

Students are given the set of straws and the nylon line and are asked to make a polygon of a given number of sides and study its properties. Also the properties of a figure obtained by superimposing two polygons are studied. Many more activities can be discovered by the students by manipulating the materials given.

The activity is extended by giving a fixed number of straws and asking the students to make all possible types of polygons.

(class 5-8)

Source: The Mathematics teacher, Vol.87, Nov. 1994.

## 6. MAKING MOLECULAR MODELS IN GEOMETRY.

We know that chemistry and geometry are connected in many ways, of which the shapes of molecules is one important overlapping area. This activity is for the students to study few such models by actually making them.

First the cubic model is created by using a sponge ball. The ball is cut into eight equal pieces and these pieces are re-assembled with their circular faces projecting inside. The pieces are connected by tooth pick or small metal wires. At this point the volume of the cube and that of the ball are compared and the volume of the empty space inside the cube is calculated. The activity can be extended by asking the students to arrange eight such cubes to get a bigger cube and study the empty space, which is actually of the shape of the original sphere (ball).

In the next part a methane molecule is constructed and the tetrahedral structure of the molecule is formed. Also a tetrahedron is prepared from straws which is dipped in soap solution to get a three dimensional figure. This figure is drawn on the paper and the property studied.

(class 8-10)

Source: The Mathematics teacher, Vol.87, Jan. 1994.

## 7. THREE AND FOUR DIMENSIONAL SPACE MODELS USING STICKS (STRAWS)

The four dimensional cube is called the 'hypercube' which actually does not exist in our three dimensional world, but a model that looks like a hypercube (or 4-cube) in 3-D projection is made as explained below.

There are two ways of making a 4-cube. One is a cube- inside a cube model and the other is an inter-connected cube model with corresponding vertices joined. Here the second method of making is explained.

First four separate cubes are constructed each of dimension 15cm x 15cm x 15cm. Further these four cubes are interlocked in such a way that the corresponding vertices are joined to the adjacent cube. This resulting cube is a 4-cube. The activity is extended for the construction of rectangular prisms.

(class 8-10)

Source: The Mathematics teacher, Vol.88, Oct. 1995.

## 8. ACTIVITY USING SPYROGRAPH.

A spyrograph is often used by the children to form different floral designs. Here we help the students to explore some very important geometrical patterns using the spyrograph and discover a few new things about the relation between the teeth of the spyrograph.

One relation that is discovered by drawing a design is  $n \times p = 2 \times m$ . Where  $n$ ,  $m$  are the teeth in the inner and outer gear used and  $p$  is the number of petals in the design. There are two more relations to be discovered,  $m \times r = \text{L.C.M} (m,n)$  and  $\text{L.C.M} (m,n) = n \times p$ , where  $r$  is the number of orbits.

The figures obtained with the spyrograph are used for explaining mirror and rotational symmetry. Further students can be asked to find out the relationship between the ratio of number of teeth and the corresponding number of petals.

(class 7-10)

Source: The Mathematics teacher, Vol.87, Nov. 1994.

## 9. ACTIVITY TO STUDY REFERENCE ANGLE.

Reference angle is a term used in trigonometry. It is the positive acute angle between the terminal ray and the X-axis. It is seen that the students get confused with this idea and use Y- axis in the place of X-axis. This activity helps to remove the

confusion. The students construct a device themselves for doing the activity using thick hard board pieces. Two pieces represent X, Y axis and the third represents the terminating ray. Each time in the activity the terminating ray is rotated and the reference angle is found.

(class 8-10)

Source: The Mathematics teacher, Vol.85, Oct. 1992.

## **10. THREE DIMENSIONAL GRAPHING.**

Three dimensional graphing presents a simple and interesting approach for the studying geometric solids. A three dimensional model is used for this purpose. The model is made using four sheets of acrylic with 20cm x 20cm dimension. The sheets have grids and the three axes are marked on them.

Using this model students are asked to draw a line in a 3-D space when the coordinates for the points are given. For representing the points stickers or small buttons can be used. This model can also be used for playing the tic-tac-toe game when a set of coordinates are given for plotting.

(class 8-10)

Source: The Mathematics teacher, Vol.85. Oct. 1992.

## **11. ACTIVITY TO IDENTIFY A POINT AND ITS QUADRANT.**

In the rectangular coordinate system exercise the students generally fail to identify the correct quadrant for a given point. In this activity students practice the method of finding the quadrant with the help of an aid. The aid is prepared from cardboard or acrylic and is divided into four quadrants by placing two slits or channels. Two wheels or buttons with spokes attached to them move freely in the slits. Here each spoke will represent the position of the coordinates of the point. The position of the point and the quadrant is got by seeing the intersection of the spokes on the plane.

Repeated exercise with this model will help the students to understand the concept of a quadrant.

(class 8-10)

Source: Nil.



## 12. ACTIVITY USING TOOTHPICK.

Toothpicks are cheap and easily available material from which a number of geometrical figures can be formed and their properties studied. It is a good medium for the exploration of geometry figures and problems and any concept can be explained using toothpicks.

In a particular class given an activity, say the study of triangles, each student is given a packet of toothpicks and asked to make triangles of different types taking 3, 4, 5 ... toothpicks at a time. The study of relationships between different figures is also explained. For the tooth pick activity it is necessary to give a set of tasks to be performed by the students in a given interval of time.

The toothpick activity can be extended to any level by suitably modifying the required conditions.

(class 6-10)

## SECTION - B: ARITHMETIC

### 1. SOLVING INEQUALITIES BY SIGN CHART METHOD.

The sign chart method explained here provides an alternate method to solve the inequalities. In this method first the inequality is rewritten to have zero on the right hand side. Then the expression is factorised and the roots are noted down. A root of factors is a key point and is represented on the number line (see illustration). A vertical line is placed at the key point which acts as the demarcation, which is actually the sign chart for the first key point. Further the same method is applied for the other key point to get the sign chart. These two factors are placed in two rows. A third row is next created for the sign chart of the product of the factors. Then by seeing where the chart is positive and where is it negative the solution is obtained.

An important advantage of the sign chart is that the solution set of any associated inequality can immediately be read from the top row. This method can be used to solve inequalities of any type.

(class 8-10)

Source: The Mathematics teacher, Vol.84, Dec-1991.

## 2. FIND ABSOLUTE VALUE BY ABSOLUTE VALUE SCALE.

In this activity a connection is established between the number line and the concept of absolute value as distance. For this we require two small strips of paper of size one inch by eleven inch, one is used as number line and the other as distance scale. The ruled portion of the scale is used to write numbers. The distance scale is similar to the number line except that the natural numbers replace the negative numbers and zero is in the middle of the natural numbers.

The first activity illustrated is  $|a - b| = |b - a|$  using the example  $|-4 - 3| = |3 - (-4)|$ . At first  $|-4 - 3|$  is represented on the strips and then  $|3 - (-4)|$  is presented. In both the cases the answer is 7 as shown on the strips. The same rule is applied to solve  $|x - 2| = 3$ .

The advantage of this method is that it can be used to rewrite the equalities or inequalities in the form of absolute value. For example  $-1 < x < 7$  be written by finding the midpoint of the interval, calculating the distance from the midpoint to the end points of the interval then writing  $x$  minus the distance from the midpoint to the end point. In the example the midpoint is 3 and if we subtract 3 from the inequality we get  $-4 < x - 3 < 4$ . So the condition is  $|x - 3| = 4$ .

This method is a good example of how activities can be used to accomodate variety in classroom teaching.

(class 9-10)

Source: The Mathematics teacher, Vol.84, Apr-1991.

## 3. STUDY OF NUMBER LINE USING SYMBOLS.

For this activity first the prime numbers are given appropriate symbols like red dot for 2, green dot for 3, blue dot for 5, orange square for 7, and so on. The selection of the symbols need not be the same as explained here. Next the number grid of first 100 numbers is prepared using these symbols.

To find the GCF of a pair of numbers in this activity, we place the symbols of the factors of the given numbers and then the common symbols are collected to get the answer. The same rule is followed in obtaining the LCM of numbers or to find the square root of a number.

Since this method provides a visual out look to the problem it is easy for the students to grasp the idea.

(class 5-8)

Source: The Mathematics Teacher, Vol.85, Sep-1992.

#### 4. GEOMETRIC PATTERNS FOR EXPONENTS.

This activity involves the physical construction of a sequence of shapes which leads to the definition of exponents and division of exponents on same base.

For this each student is supplied with three sheets of inch grid paper and two or three pieces of coloured paper. To begin the activity first a 1" x 2" rectangle is cut out from the inch grid paper and the area is labeled as 2. Further this paper is placed on another grid paper and a 2" x 2" shape is created. The same method is followed to get a 2 x 4 rectangle and which is labeled as  $2 \times 2 \times 2$ . Also a  $2 \times 2 \times 2 \times 2$  block is created by the same method. all these shapes are then pasted on the colour paper to get a sequence of powers of 2. It is also possible to create the grids having zero or negative powers of 2. Then the students are asked to study the properties of the series and find quotients by diagram.

(class 5-8)

Source: The Mathematics teacher, Vol.85, Dec-1992.

#### 5. INEQUALITIES, ABSOLUTE VALUE ON NUMBER-LINE.

The concept of absolute value is more often confused by the students due to the solution of problems that combine equalities and absolute value. This activity involves consideration of the function concept as the graph created by associated ordered pairs rather than functional values.

First of all the ordered pairs of real numbers such that the first and the second numbers are additive inverses of each other are selected. Then each pair is connected to the non-negative member of the pair. This correspondence is a function from  $\mathbb{R}$  to  $\mathbb{R}^2$  to  $\mathbb{R}$ .

In the next stage the absolute function is graphed using a horizontal line and a vertical line whose initial point is located at the zero point on the line.

For solving the inequality  $|2x + 3| = 7$ , we first locate 7 on the vertical line and join it to the two possible values in the problem i.e. -7 and 7. Since a real number cannot be located at two distinct places, the students arrive at a solution that  $2x + 3 = 7$  or  $2x + 3 = -7$  and thus solve the equations to get the answers.

This pictorial approach will help the students to gain confidence in the conventional approach that is followed at later stages.

(class 6-8)

Source: The Mathematics Teacher, Vol.85, Dec-1992.

## **6. MULTIPLYING OF INTEGERS BY HIP-HOP GAME.**

In the middle school it is seen that the students are made to learn the multiplication table by imposition. Here is a simple game which enables the students to learn the multiplication just by playing.

This game also called the froggy-frolic game has a set of hop cards with frog and hop written on them. The frog stands for the number to be multiplied and hop stands for the number of times it to be multiplied. For example frog(-3), hop(4) means -3 is to be multiplied 4 times and the product is noted down to make moves on the game board. the game is won if the frog jumps into the pond or the swamp.

(class 5-7)

Source: The Mathematics teacher, Vol.85, Jan-1992.

## **7. OPERATION ON DECIMALS.**

This activity provides an opportunity for students to explore the effect of addition, subtraction, multiplication and division on decimal numbers. For this activity a maze board is supplied and the students are at find a path to reach the end by doing the operations told.

Students are asked to find out the shortest path with highest end value or lowest end value or the path which gives an answer near to the starting number. Calculators are not allowed and wherever non-terminating decimals come then the answer is to be taken for two decimal places only.

(class 6-9)

Source: The Addenda Series. NCTM.

## **8. ACTIVITY FOR LAGRANGE'S THEOREM.**

According to Lagrange's theorem, every whole number can be decomposed into sums of no more than four squares. To verify this concept in this activity students are given a set of numbers and asked to split them into the square numbers. For

example  $97 = 81 + 16$  or  $97 = 64 + 25 + 4 + 4$  where all the numbers are square numbers.

The students can be given application problems that use this idea to solve. One such problem is to get a square carpet for a room of dimension 20 square meter.

(class 6-10)

Source: The Most Beautiful Mathematical Formulas, by Lionel..etl.

## **9. MULTIPLICATION OF DECIMALS BY CONNECTION.**

The process of multiplication is not so easy to perform. This activity provides a unique method of multiplying two decimal numbers.

The two numbers to be multiplied are selected by two students. Then on a grid paper of 10 by 10, each student shades the region equal to the decimal number given exactly on the part of the grid paper. Different colours are used by the students such that the overlapping region is easily recognised. The mixed part is again a part of the bigger of the two decimals used in the multiplication, and hence it gives the product. For example in the product of .5 and 2.6, one student colours for the 2.6 part of the grid and the other for .5 of the grid such that she uses the already coloured part also. So the overlapping colour region is now .5 of 2.6. Thus the product of .5 x 2.6 is 1.30.

This method can extensively be used for teaching decimal multiplication for the under-achieving student.

(class 5-8)

Source; Addison-Wesley Mathematics, grade-6 (27600)

## **10. FORMULA CRISS-CROSS, A MENSURATION PUZZLE.**

In the mensuration chapter of arithmetic there are a number of formulae which the students often forget. In this cross-word puzzle an attempt is made to reinforce these formulae. The crossword puzzle has instructions such as the following:

### **TOP TO BOTTOM**

1. I will be the distance covered by you when you run around a circle.
2. I will help you to find the section of the pizza that you ate last night.

3. How much barbed wire do you need to protect your square garden from stray cattle ?
4. How do you find the amount of roofing needed to cover the earth from a meteor shower ?
5. What is the area of the top and bottom of a cylindrical drum ?
6. How much water is there in the rectangular tank at the top of your house ?
7. Do you know the area of the ice cream cone ?
8. How much air do you need to fully inflate the football bladder ?
9. What is the area of the parallelogram of sides  $a$ ,  $b$  units ?

### **LEFT TO RIGHT**

1. Do you know how much milk you drink in an ordinary glass ?
2. What is the area of a half circle ?
3. When the roller rolls once, how much area does it flatten ?
4. What is the area that you get to sleep over a disc if you are asked to do so ?
5. How much air does the joker fill in his conical hat ?
6. The custard that you ate just now in the bowl, how much was that ?
7. How do you find the area of a right triangle?
8. It is the area of an equilateral triangle.

Source: nil.

## SECTION - C: ALGEBRA, TRIGNOMETRY AND PROBABILITY

### 1. A VISUAL LOOK FOR POLYNOMIALS.

The chapter of polynomials introduced in the middle school level is a little difficult for the students to understand as they are learning algebra for the first time. So in this activity the visual approach is used for explaining the kinds of polynomials. Here we use small tiles for representing the monomials, light colored tiles for positive, and dark ones for negative. Depending upon the monomial the size of the tile is selected.

In the representation of a polynomial these tiles are rearranged. In this activity the students themselves arrange the tile for representing the given polynomial. By this visual method it is possible to give a clear idea of polynomials to the students.

(class 6-8)

Source: Addison-Wesley Pre-Algebra.

### 2. SOLVING EQUATIONS USING OBJECTS.

An equation is a mathematical sentence stating that the two expressions name the same number. In the solution of an equation a student is supposed to rearrange the terms to get the required answer. This is where they get mixed up and end with wrong solutions. This activity helps them to practice the solution using objects.

In this activity different kinds of colored chips are used, the circular dark ones are for adding 1 and the light ones are for subtracting 1. The square chips are for the unknown quantity, say  $x$ . A tray with a partition is used to perform the activity. In the solution of the equation the operation of adding the chips is done till they get canceled out. The remainder is the answer for the equation.

Since this method involves learning by doing, students easily learn the process.

(class 6-8)

Source: Addison-Wesley Mathematics, grade-7 (27700).

### 3. UNDERSTANDING THE IDENTITY BY PICTURES.

In algebra, we have two very important identities introduced for the students,  $a^2 + 2ab + b^2 = (a + b) \times (a + b)$  and  $a^2 - 2ab + b^2 = (a - b) \times (a - b)$ . These two are often memorized by the students without understanding. Here is a very simple method

to understand these identities. On any square paper of side  $a+b$  a floral design is drawn. The square is cut into four pieces to get two squares and two rectangles of area  $a^2$ ,  $b^2$  and  $ab$  and  $ab$  respectively. Now all the pieces intact will represent the product of  $(a + b) \times (a + b)$  and the pieces separated will represent  $a^2 + 2ab + b^2$ . The same method is used for explaining the other identities as well.

This method will definitely attract students in the classroom and hence help them learn the concepts.

(class 6-8)

Source: The Most Beautiful Mathematical Formulas, by Lionel..etc.

#### 4. PLAY BINGO TO LEARN ALGEBRA.

In this activity the game Bingo is applied for teaching some algebraic concepts. This game can be constructed by the teacher in any chapter as a revision exercise.

In this particular section, bingo is used to find the slope and intercept for a given set of equations. This game has a set of question cards and answer grids. Each student selects an answer card, and when the teacher gives out a clue to a question, the student strikes out the answer if it is in the card. The first student who exhausts all the numbers in the answer grid shouts "bingo" and is the winner. This kind of play helps in reinforcing many concepts.

(class 6-8)

Source: The Mathematics Teacher, Vol.84, May-1991.

#### 5. GRAPHICAL SOLUTION TO QUADRATIC EQUATIONS.

Students at the high school level know the traditional method of solving quadratic equations. In this activity an alternate approach for solving quadratic equations is explained.

The basic procedure used in this method requires that the students first find the line of symmetry and the vertex of the parabola. These values are then applied for getting the solution to the given quadratic equation. First the students are explained about the graph of  $y = ax^2$ , then  $y = ax^2 + bx$  and finally  $y = ax^2 + bx + c$ , which is the equation in the quadratic form.

(class 9-10)

Source: The Mathematics Teacher, Vol.89, Jan-1996.



## **6. A VISUAL APPROACH TO ALGEBRA CONCEPTS.**

This activity is an avenue for mental arithmetic and provides an opportunity to visually explore some fundamental algebraic concepts.

In this method the different numbers or variables used are given some symbol and the necessary operation is carried out. There is no particular symbol assigned for a number and it can be done in the classroom only while teaching. This visual approach help the students to grasp the concepts easily. Another added advantage of this method is that given a visual representation with the clear meaning to each symbols, students can be asked to give the mathematical equation. The different properties such as commutativity, associativity, distributivity are easily explained by this method.

(class 5-8)

Source: The Mathematics Teacher, Vol.85, Sep-1992.

## **7. A GEOMETRIC APPROACH TO TRIGNOMETRIC RATIOS.**

The sine and cosine ratios are often visualized as the lengths of the projections of a unit segment on the x- and y-axis. In the figure presented here this type of pictorial approach is adopted for four other trigonometric ratios. Using these kinds of pictures as the basis of understanding trigonometric definitions and relationships can be helpful to both the beginners and the advanced learners.

(class 9-12)

Source: The Mathematics Teacher, Vol.86, Sep-1993.

## **8. EXPERIMENTS WITH DICE.**

This is an activity to find out the number of chances of an event occurring using one die or two dice. This activity helps the students to learn the concept of probability of a given event. Students are asked to roll the dice five times, ten times, twenty times and each time note down the probability of the occurrence of some number on the dice. They can even be asked to predict the probability and then verify by experiment.

This activity can be given to introduce the probability lesson.

(class 6-8)

Source: The Mathematics Teacher, Vol.84, Apr-1991.

## 9. A PROBABILITY GAME.

This game aims at motivating the students to practise basic probability concepts and algorithms. It consists of a game board and a set of question cards. The rules of the game are as follows.

1. The player with the highest value on the die will start the game.
2. The second player now asks the question to this player. If the answer is told correctly then the player will roll the die and corresponding to the number got the move is made. If the question is answered incorrectly then the next player gets the chance.
3. Now the first player asks the question to the second player and the play continues in the same manner.
4. The question cards are picked by the player from the deck.
5. If a player gives the wrong answer then both players should discuss the solution and then verify it.
6. The student who completes the round once is the winner.

This game is to be used to supplement the classroom teaching.  
(class 6-10)

Source: The Mathematics Teacher, Vol.84, Apr-1991.

## 10. PLINKO, PROBABILITY AND PASCAL.

This activity is developed to teach Probability and Pascal triangle. For this activity we need a plinko board. A plinko board is a board having a maze of nails. In the activity the board is held at a slant and a student is asked to drop marbles from one side of the board such that the marbles roll to the other side. Each student is given 50 marbles to perform the activity. At the end he/she will find out the number of marbles in each column and draws a curve. The activity is performed on different boards having different number of nails which helps the students to find out the encounter that a marble faces in each event.

From this experiment the students are asked to find the relationship of probability and Pascal triangle. Even the role of binomial theorem can be discussed by this

experiment with some modifications on the board (we need a fixed width for the board).

(class 6-10)

Source: The Mathematics Teacher, Vol.88, Apr-1995.

## **11. A FUNCTION MACHINE.**

Function is a relation between the elements of two non-empty sets. In the middle and high school level the students are supposed to find the function when the sets are given. To make this process interesting in the class we can use the function machine.

A function machine is a small box which has an input opening, an output opening and a screen. During the activity for the student the input and the output are shown and then they are asked to write the function described on the screen for input and output. This will help the students to discover the function for any sequence.

(class 6-8)

Source: Addison-Wesley Mathematics, grade-4 (27400)

## **12. VISUALISING FUNCTION BY PLAYING BLOCKS.**

The concept of function here is explained with spatial visualization activities. For this activity each student is given ten blocks each of which is a 1 cm cube. Then the students are asked to arrange these blocks in different ways but without exceeding a 4cm x 4cm x 2cm frame.

The students practise drawing 2-D views of the arrangement: topview, front view and side view. The convention adopted is that the top view will also show depth in each square cm. Hence every top view uniquely determines an arrangement. The side-views do not always uniquely determine an arrangement, even if the front and side views are taken together.

In the next stage students establish a function between the top view and the two side views. Thus this activity provides a simple exercise to the students to learn the concept of function. The activity can be extended by giving the function and asking students to form the arrangement of blocks.

(class 6-8)

Source: The Mathematics Teacher, Vol.85, Nov-1992.

### **13. INVERSE OF A FUNCTION.**

Students in a classroom usually find the topic of inverse functions difficult. For this purpose a set of activities for the students is provided here. All these activities contain examples from everyday life as well as purely algebraic compositions.

In each activity first the function is drawn on a transparency, then the 45 degree line between the x and y-axis is plotted. Further the transparency is turned down and the students are asked to find out the inverse of the given function.

(class 6-10)

Source: The Mathematics Teacher, Vol.89, Feb-1996.

## **SECTION D: MISCELLANEOUS**

### **1. PUZZLES, GAMES, ENRICHMENT PROBLEMS.**

We know that general classroom teaching of mathematics is often dry due to lack of supplementary materials in the form of games, activities, puzzles etc. In order to help in this regard a series of such activities is suggested for the use of the teacher. These activities cover a wide range of topics and can be used by the teacher at any point of time.

For every activity a set of instructions are given at the beginning and the teacher should make sure that the students understand these instructions before taking up the activity.

(class 5-10)

Source: Intermediate G.C.S.E. by D.Rayner.

### **2. ACTIVITY TO MAKE FLEXAGONS.**

Flexagons are nothing but hexagons made from paper strips that show an unusual property when pushed in or pulled out at different places.

The activity describes how to make a flexagon from a strip of paper divided into 19 equilateral triangles. It suggests colouring of the flexagons to show the change in faces as they are flexed more appropriately.

(class 6-10)

Source: The Mathematics Teacher, Vol-84, Nov-1991.

### **3. LEARN MATHS BY WEIGHING.**

The similarity or congruence between different geometrical figures can be verified by comparing their weights. In this activity an opportunity is given for the students to discover the relationship between weight of an object to its area.

For performing this activity templates are used. A template is a thin sheet of object made of any material. But the templates used for an activity must of the same material. The templates are cut into required shapes and weighed. Now since the density of the material is same their area are compared to arrive at the solution. For performing this activity in the classroom we can use the card board pieces. In order to avoid the wastage of time in cutting out the necessary shapes, we can provide a set of ready made templates.

Few activities listed are the ratio of area of a circle to that of the cycloid of the same radius, the lune of Hippocrates to that of a right triangle. The activities can be designed by the teacher depending upon the need of the topic.

(class 6-10)

Source: The Mathematics Teacher, Vol.85, May-1992.

### **4. ACTIVITY ON A MARBLE SIFTER.**

The marble sifter activity is developed to teach the students the concept of half life of an isotope. It uses the idea of probability in arriving at the solution.

A marble sifter consists of a box with a sifter arranged at the top which sifts up and down. The activity starts with one hundred marbles placed on the sifter. Now a sift is given to the marbles so that few marbles go down. Students are asked to note down the remaining marbles. The activity is repeated and each time the no. of left over marbles are counted. The results are then tabulated and discussed in the class to arrive at the definition of half life of an isotope.

(class 8-10)

Source: The Mathematics Teacher, Vol.88, Dec-1993.

### **5. SOLOMON'S GAME AND NINE MEN'S MORRIS.**

In this section the two games described help the students to develop critical thinking abilities.

The first game is Solomon's game, a game attributed to King Solomon by Martin Gardner, the famous popular mathematics writer. However the game was probably

invented by Gardner himself. This game is similar to checkers and involves capturing as many of the opponents pieces as possible. The board is shaped like a six pointed star composed of two equilateral triangles, one of them inverted. The game is popular with students and reinforces reasoning skills.

The second game is Nine Men's Morris, a game which was popular in medieval England and France. This game shares its origin with ancient versions of tic-tac-toe and calls for arranging three pieces in a line. But it also involves elements of checkers like capturing an opponents pieces. This game is also enjoyable and gives students an opportunity to exercise reasoning skills.

(class 6-10)

Source: The Mathematics Teacher, Vol.85, Feb-1992.

## **6. PROPERTY OF CENTRAL POLYGONAL NUMBERS.**

The activity on central polygonal numbers is an extension of the study of polygonal numbers. The classical polygonal numbers are formed by expanding from a vertex the polygon thus getting a similar dot pattern. The centrally generated patterns can also be developed.

The study of these polygonal numbers will help the students to develop the concept of recursion, recursive sequences, and recurrence relations. A set of activities for reference is given in the appendix.

These activities can be extended to solve the closed formulas expressed as quadratic functions. That is the students must find  $a, b$  in the function  $f(n) = an^2 + bn + c$  by suitably substituting for  $a, b$ . (class 9-10)

Source: The Mathematics Teacher, Vol.84, Dec-1991.

## **7. SOME COLOURFUL MATHEMATICS.**

Coloring problems are not common in our school curriculum, but they have an important place in mathematics. The most important problem is the four color problem. It states that the minimum number of colors needed to paint the points of the plane, such that no two points one inch apart are of the same color, is at least four but no more than seven.

In the beginning of the activity the students are asked to color the closed curves with two different colors, then the three color problem is explained and further the four color problem. For students to practise, a set of activity sheets are enclosed in the appendix. These activities can be given as enrichment to the students.

(class 6-10)

Source: The Mathematics Teacher, Vol-89, Apr-1996.